ENGINEERING MECHANICS

Dept. of Civil Engineering
B. Tech I Year II Semester
ENGINEERING MECHANICS: 23CE101

Course Objectives

- To get familiarized with different types of force systems.
- To draw accurate free body diagrams representing forces and moments acting on a body to analyze the equilibrium of system of forces.
- To teach the basic principles of center of gravity, centroid and moment of inertia and determine them for different simple and composite bodies.
- To apply the Work-Energy method to particle motion.
- To understand the kinematics and kinetics of translational and rotational motion of rigid bodies.

UNIT I 9 Hours

Introduction to Engineering Mechanics Basic Concepts. Scope and Applications Systems of Forces: Coplanar Concurrent Forces Components in Space Resultant Moment of Force and its Application Couples and Resultant of Force Systems. Friction: Introduction, limiting friction and impeding motion, Coulomb's laws of dry friction, coefficient of friction, Cone of Static friction.

UNIT II 9 Hours

Equilibrium of Systems of Forces Equilibrium of Coplanar Systems, Graphical method for the equilibrium, Triangle law of forces, converse of the law of polygon of forces condition of equilibrium, Equations of Equilibrium for Spatial System of forces, Numerical examples on spatial system of forces using vector approach, Analysis of plane trusses.

UNIT III 9 Hours

Centroid: Centroids of simple figures (from basic principles) 9 hours Centroids of Composite Figures. Centre of Gravity: Centre of gravity of simple body (from basic principles), Centre of gravity of composite bodies, Pappus theorems.

Area Moments of Inertia: Definition Polar Moment of Inertia, Transfer Theorem, Moments of Inertia of Composite Figures, Products of Inertia, Transfer Formula for Product of Inertia. Mass Moment of Inertia: Moment of Inertia of Masses, Transfer Formula for Mass Moments of Inertia, Mass Moment of Inertia of composite bodies.

UNIT IV 9 hours

Rectilinear and Curvilinear motion of a particle: Kinematics and Kinetics - Work Energy method and applications to particle motion-Impulse Momentum method.

UNIT V 9 hours

Rigid body Motion: Kinematics and Kinetics of translation, Rotation about fixed axis and plane motion, Work Energy method and Impulse Momentum method.

UNIT 1

- Introduction to Engineering Mechanics: Basic Concepts.
- Scope and Applications Systems of Forces: Coplanar Concurrent Forces, Components in Space, Resultant, Moment of Force and its Application, Couples and Resultant of Force Systems.
- Friction: Introduction, limiting friction and impeding motion, Coulomb's laws of dry friction, coefficient of friction, Cone of Static friction.

INTRODUCTION TO ENGINEERING MECHANICS

Mechanics

Mechanics is a branch of physical science that deals with state of rest or motion of physical object which is subjected to action of forces.

Mechanics :— Rigid body mechanics
Deformable body Mechanics
Fluid mechanics

Rigid body Mechanics :- Statics
Dynamics

Statics :- Statics deals with the equilibrium of bodies, means a body either in rest (or) move with a constant velocity. (Acceleration is Zero)

Dynamics :- Accelerated motion of bodies.

Fundamental concepts

length (L) — metre | Displacement

time (t) — Sec, minute, hours

Mass (m) — kg, lb

Force (F) — N, kN
- Push
- Pull

Particle

Rigid body

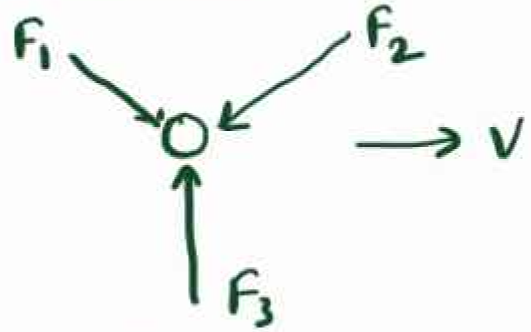
Concentrated force

Continuum:

A body consists of a continuous distribution of matter. The body is treated as continuum.

Newton's law of motion

First law



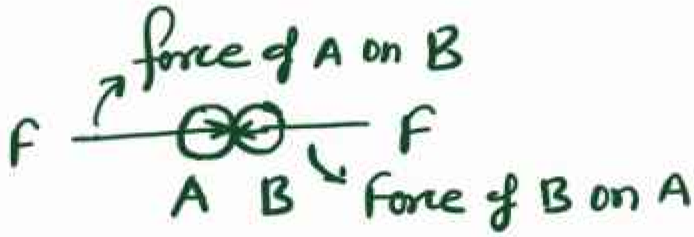
Equilibrium

Second law

$$F = ma$$

$$F \longrightarrow O \quad \rightarrow a$$

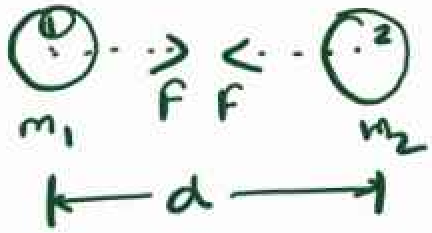
Third law



Action- reaction

Weight $W = mg$

Newton's law of gravitation



$$F = G.\frac{m_1 m_2}{d^2}$$

velocity (m/sec) — Rate of change of displacement w.r.t. time.

acceleration (m/s^2) - Rate of change of velocity w.r.t. time.

$$a = \frac{dv}{dt}$$

$g = 9.81\ m/s^2$

$1\ kg = 9.81\ N$

$10\ kg = 98.10\ N$

momentum:

momentum = Mass x Velocity

Conversion

nano — 10^{-9}

micro(μ) — 10^{-6}

milli (m) — 10^{-3}

kilo (k) — 10^{3}

mega (M) — 10^6
giga (G) - 10^9

Convert 2Km/h to m/sec

Scalars and Vectors

Scalars: Compleately specified by its magnitude.
Ex. length, mass, time
Vector: Physical quantity that requires both a magnitude and a direction for its compleate description.

Force, position, moment.

Dimensions:

$$a = m/s^2 = \frac{L}{T^2} = LT^{-2}$$

$$F = ma = \frac{ML}{T^2} = MLT^{-2}$$

Velocity
Momentum
Area
Volume

Force:

From Newton's 2^{nd} law of motion, unit for can be defined as the force required to produce unit acceleration of 1kg of mass.

It is noted that a force is completely specified only when the following four characteristics are specified.

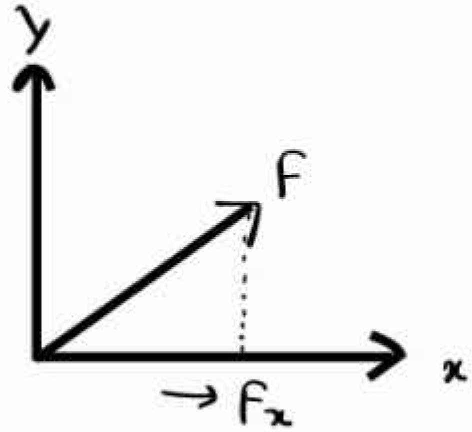
- Magnitude
- Point of application
- Line of action
- Direction.

System of forces

When several forces act simultaneously on a body, they constitute a system of forces.

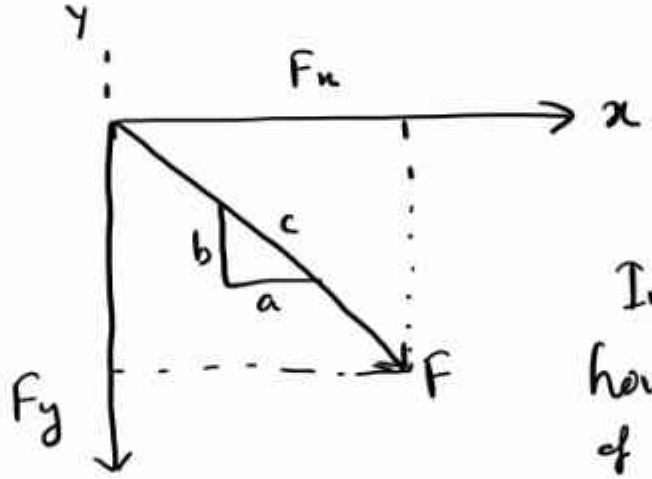
Coplanar force system:

If all the forces in a system lie in a single plane.



When a force is resolved into two components along the x and y axis, the component are then called rectangular component.

$$F_x = F\cos\theta \quad \& \quad F_y = F\sin\theta$$



Instead of using θ, however, the direction of F can also be defined using a small "slope".

$$\frac{F_x}{F} = \frac{a}{c}$$

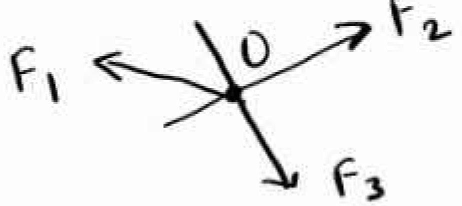
$$F_x = F\left(\frac{a}{c}\right)$$

& $$\frac{F_y}{F} = \frac{b}{c}$$

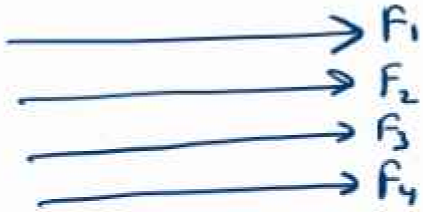
$$F_y = F\left(\frac{b}{c}\right) \text{ (or) } -F\left(\frac{b}{c}\right)$$

Concurrent force system

If the line of action of all the forces in a system pass through a single point, then it is called a concurrent force system.



Parallel force system



Collinear force system

If line of action of all the forces lie along a single line then it is called a collinear force system.

Coplanar concurrent forces

line of action of all forces pass through a single point and forces lies in a same plane.

Coplanar non-concurrent forces

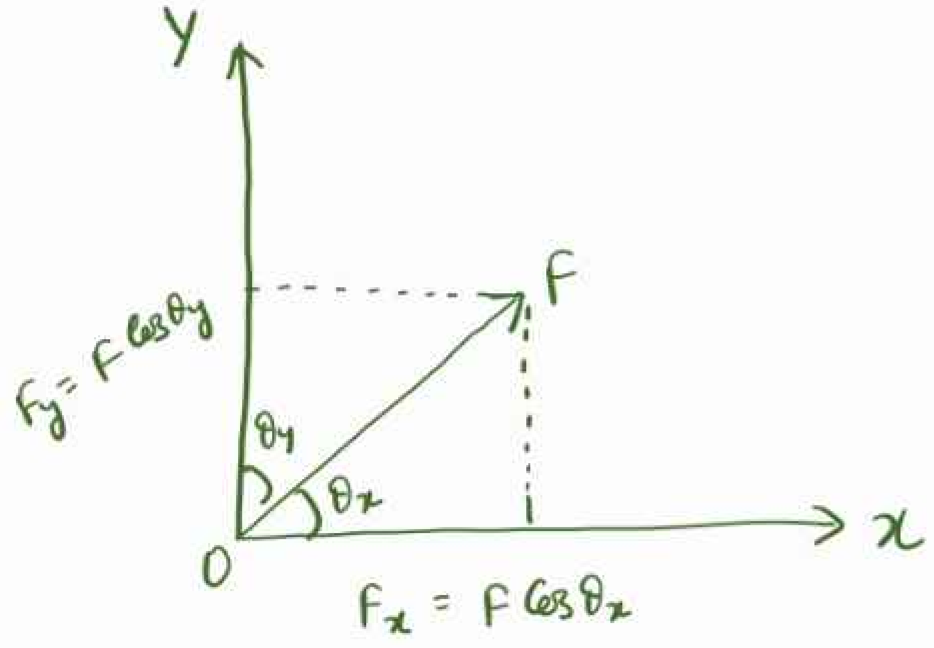
All forces do not meet at a point but lie in a single plane.

Non-coplanar parallel forces – Not in same plane.

Non-coplanar concurrent forces : All forces do not lie in a same plane, but their line of action pass through a single line.

Non-coplanar non concurrent forces

Vector Representation of forces and moments



To present clear direction, the vector is represented by its components in the cartesian coordinates.

$$F = F_x + F_y$$

More convenient way for handling vectors is to represent in terms of unit vectors in cartesian coordinate directions. If i and j are the unit vectors in the coordinate directions x and y.

$$F_x = F_x i$$

$$\& \quad F_y = F_y j$$

where F_x & F_y represent only magnitudes.

$$F = F_x i + F_y j$$

$$F_x = F \cos\theta_x \quad \& \quad F_y = F \cos\theta_y$$

$$F = F \cos\theta_x i + F \cos\theta_y j$$

$$F = F(l i + m j)$$

Magnitude of forces

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(Fl)^2 + (Fm)^2}$$

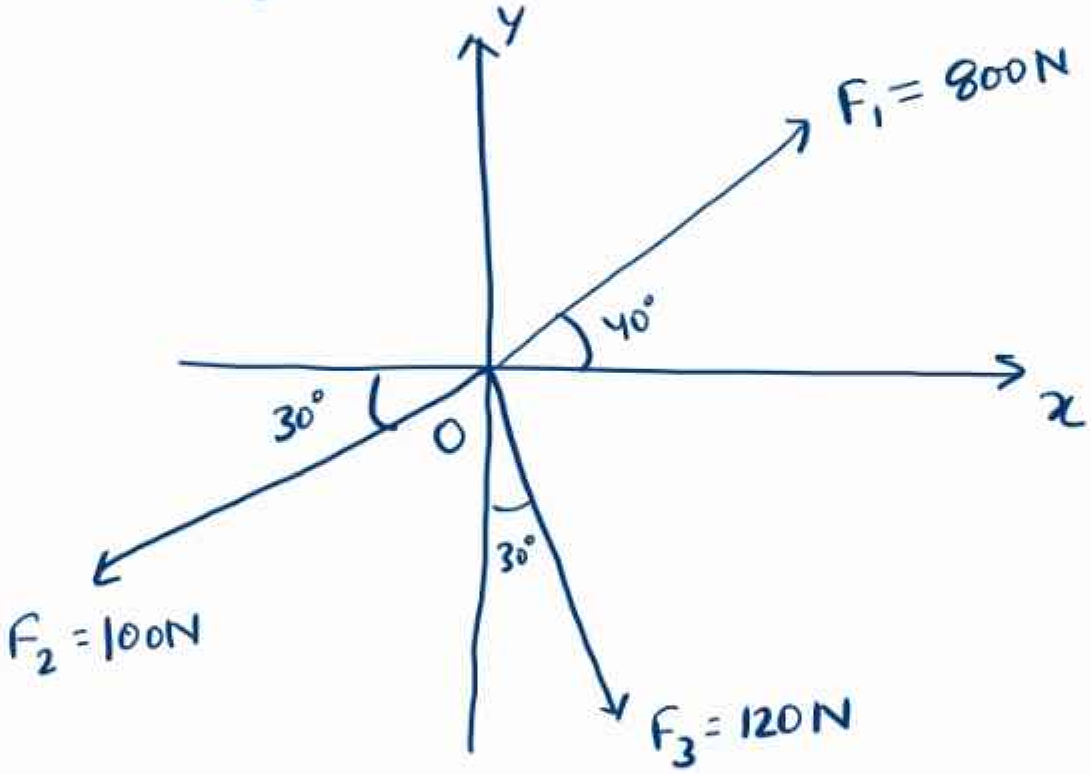
$$F = F\sqrt{l^2 + m^2}$$

Similarly vector in three dimensions.

$$F = F_x + F_y + F_z$$

$$= F_x i + F_y j + F_z k$$

Find the vectorial form of forces for F_1, F_2 & F_3 shown in figure.



Solⁿ. For force F_1 = Magnitude 800 N

$$\theta_x = 40°$$

$$F_{1x} = 800 \cos 40° \quad , \quad F_{y1} = 800 \sin 40°$$

$$= 612.84 \text{ N} \qquad\qquad = 514.23 \text{ N}$$

$$\boxed{F_1 = 612.84\, i + 514.23\, j}$$

For force F_2 : Magnitude $F_2 = 100\,N$ $\theta_2 = 30°$

$$F_{2x} = -100 \cos 30° = -86.60\,N$$

$$F_{2y} = -100 \sin 30° = -50\,N$$

$$F_2 = F_{2x} + F_{2y} = -86.60i - 50j$$

$$\boxed{F_2 = -86.60i - 50j}$$

For force F_3 magnitude $F_3 = 120\,N$, $\theta_3 = 30°$

$$F_{3y} = -120 \cos 30° = -103.92\,N$$

$$F_{3x} = 120 \sin 30° = 60\,N$$

$$\boxed{F_3 = 60i - 103.92j}$$

Position vector

Addition of vector

Subtraction of vectors

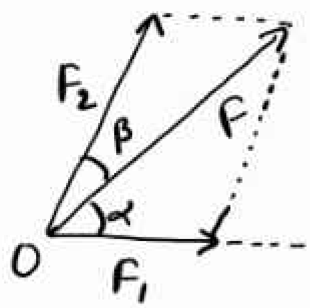
Displacement vector

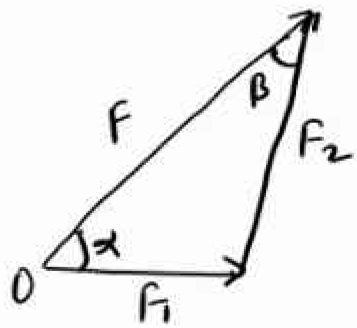
Dot Product of vector

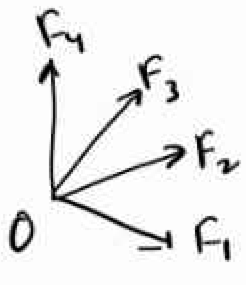
Cross project of vector

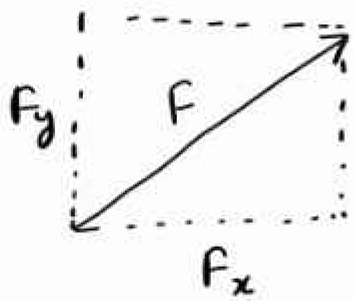
Resolution of concurrent coplanar forces

Resolution of forces is the process of finding a number of component forces which will have the same effect on the body as the given single force.









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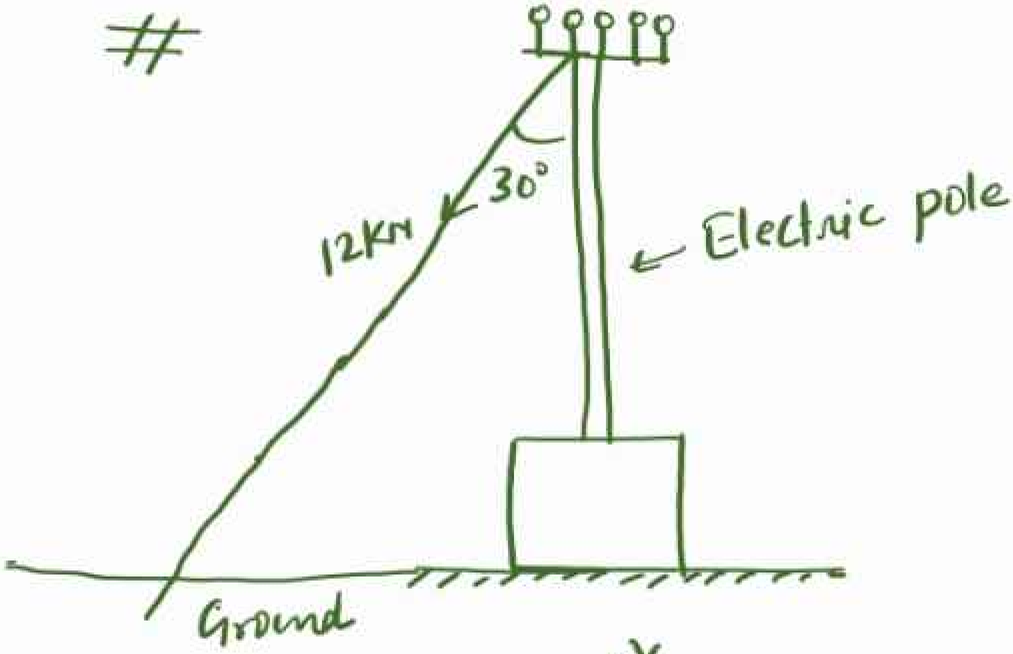

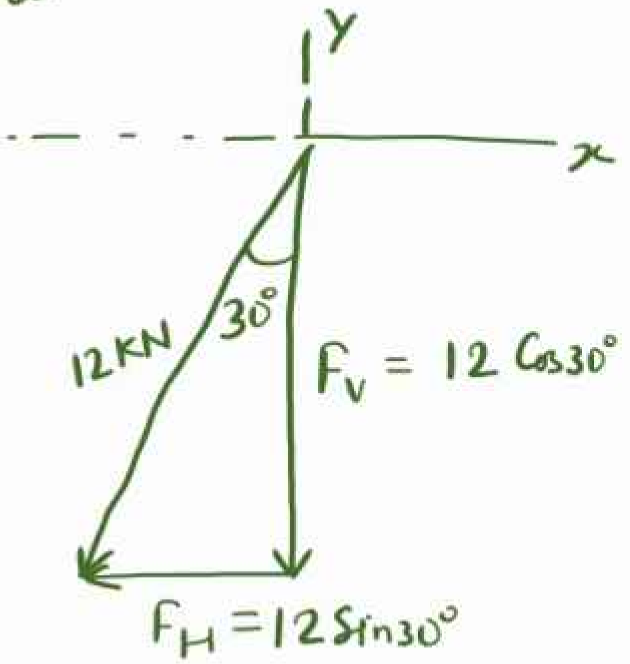

$F_V = 12 \cos 30° = 10.392$ kN (downward)

$F_H = 12 \sin 30° = 6$ KN (towards left)

Since the coordinates selected are as shown as, taking i as unit vector in x-direction and j as unit vector in y-direction.

$F_x = -6i$ and $F_y = -10.392j$

$F = -6i - 10.392j$

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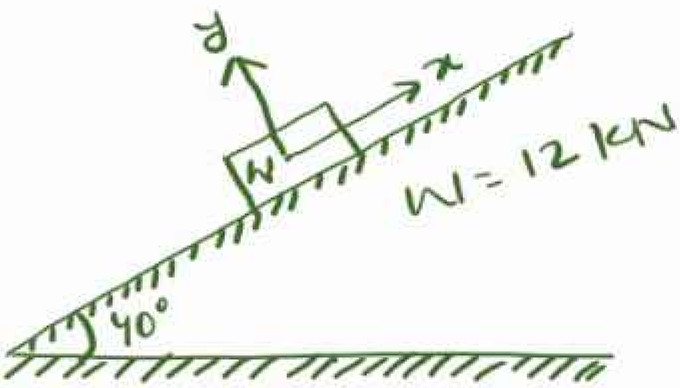
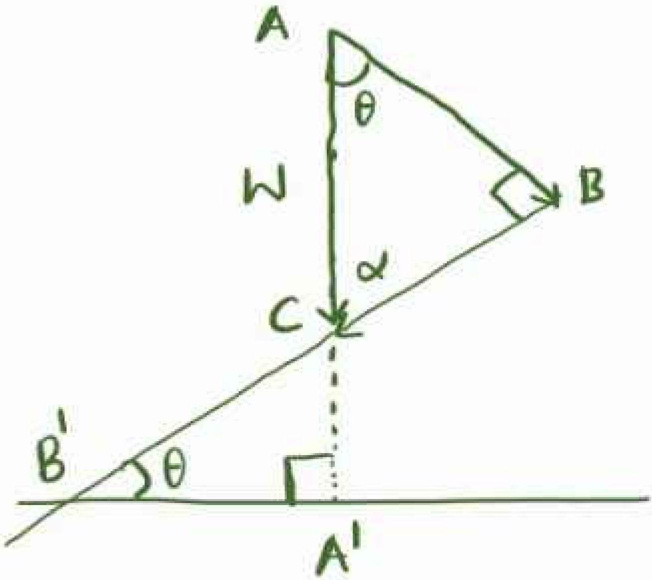
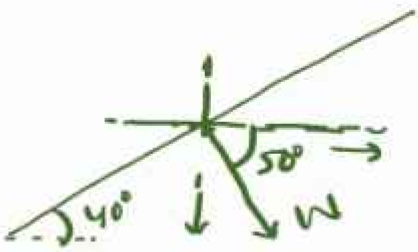




$\triangle ABC \;|||\; A'B'C$ since, $\angle ACB = \angle A'CB'$

and $\angle ABC = \angle CA'B' = 90^\circ$

$\therefore \angle BAC = \angle A'B'C = \theta = 40^\circ$



$F_x = W\cos 50^\circ$

$F_y = W\sin 50^\circ$

Here, $W = 12\ KN$

$$F_x = W\cos 50^\circ = 12\cos 50^\circ = 7.713\ KN$$

$$F_y = -W\sin 50^\circ = -12\sin 50^\circ = -9.193\ KN$$

$$F = 7.713\,i - 9.193\,j$$

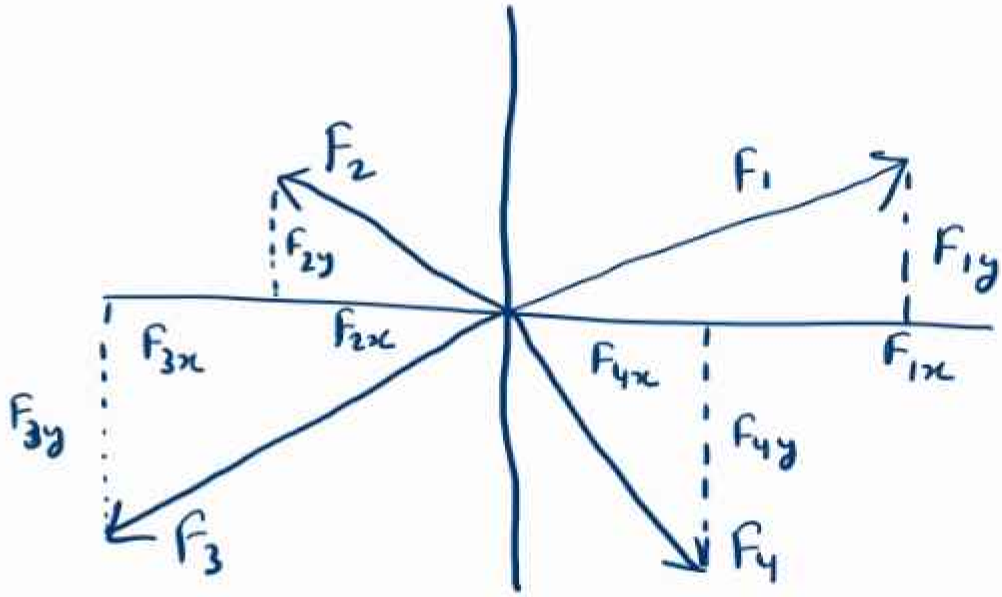
$$F_R = \sqrt{(7.713)^2 + (9.193)^2} = 12\ KN$$

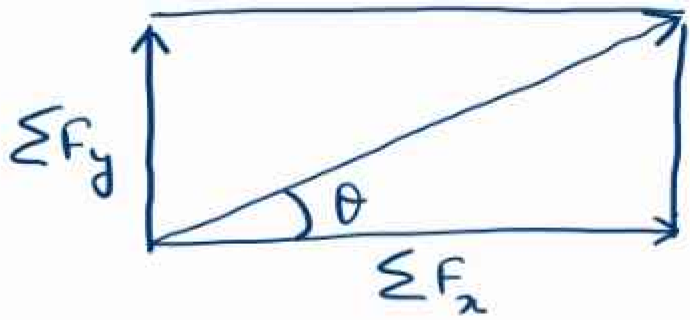
Composition of concurrent coplanar forces

It is possible to find a single force which will have the same effect as that of a number of forces acting on a particle. Such single force is called as Resultant force and the process of finding resultant force is called Composition of forces.

This can be found by using Parallelogram law, triangle law and polygon law.

In analytical method the component of each forces in the coordinate directions are found and they are expressed in the form of vectors.





If F_1, F_2, F_3, F_4 are forces in the system, then it can be written as

$$F_1 = F_{1x} i + F_{1y} j$$

$$F_2 = F_{2x}i + F_{2y}j$$

$$F_3 = F_{3x}i + F_{3y}j$$

$$F_4 = F_{4x}i + F_{4y}j$$

In the above expression, F_{2x}, F_{3x}, F_{3y} & F_{4y} will have negative values.

Resultant, $R = F_1 + F_2 + F_3 + F_4$

$$= F_{1x}i + F_{1y}j + F_{2x}i + F_{2y}j + F_{3x}i + F_{3y}j + F_{4x}i + F_{4y}j$$

$$R = R_x i + R_y j$$

Where, $R_x = \Sigma F_x$ & $R_y = \Sigma F_y$

$$R = \Sigma F_x i + \Sigma F_y j$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

If θ is angle b/w x-axis and the force

$$\boxed{\tan\theta = \frac{\sum F_y}{\sum F_x}}$$

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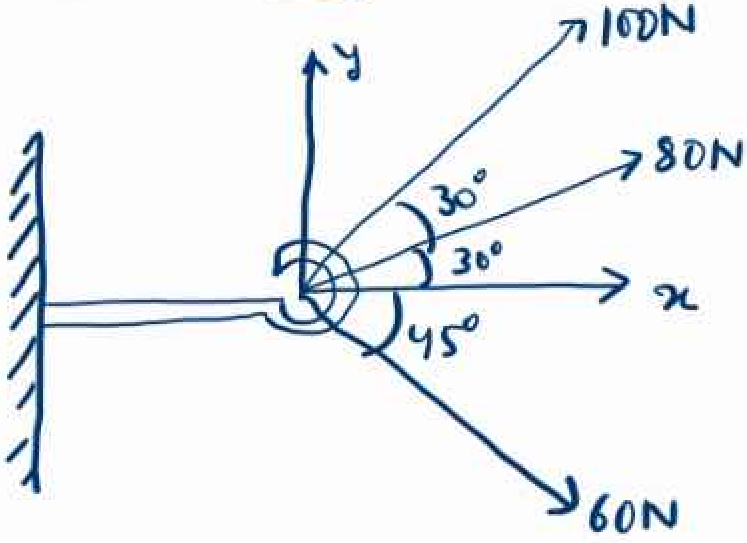
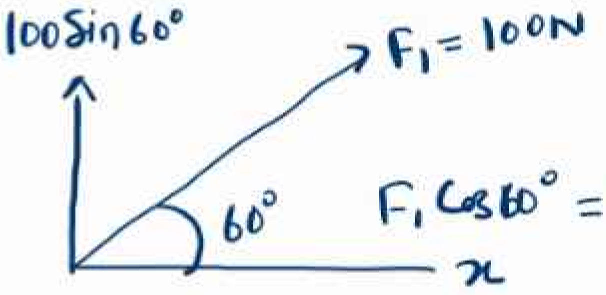


Determine the resultant of three forces acting on a hook as shown in figure.

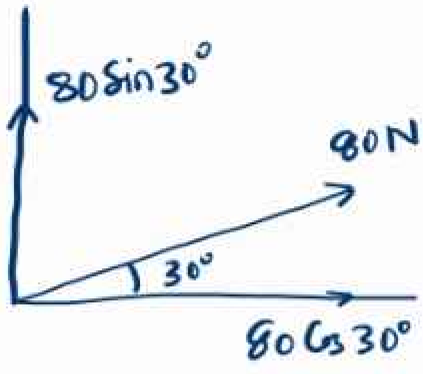
$F_1 = 100N$

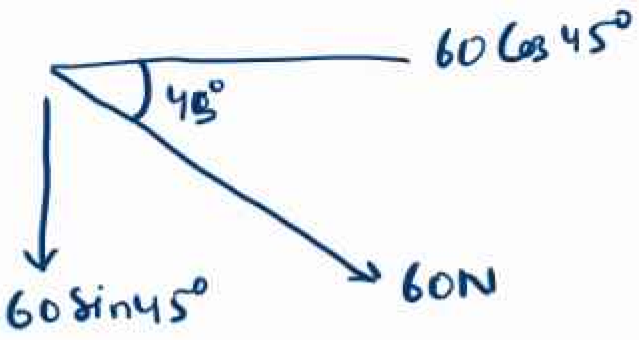
$F_2 = 80N$

$F_3 = 60N$



$F_1 \cos 60° = 100 \cos 60°$





$$F_1 = (100\cos 60^\circ)\, i + (100\sin 60^\circ)\, j$$

$$F_2 = (80\cos 30^\circ)\, i + (80\sin 30^\circ)\, j$$

$$F_3 = (60\cos 45^\circ)\, i - (60\sin 45^\circ)\, j$$

$$R = F_1 + F_2 + F_3$$

$$= (100\cos 60^\circ + 80\cos 30^\circ + 60\cos 45^\circ)\, i + (100\sin 60^\circ + 80\sin 30^\circ - 60\sin 45^\circ)\, j$$

$$= 161.71\, i + 84.17\, j$$

$$R = \sqrt{(161.71)^2 + (84.17)^2} = 182.3\text{ N}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{84.17}{161.71} = 0.5205$$

$\theta = 27.5^\circ$

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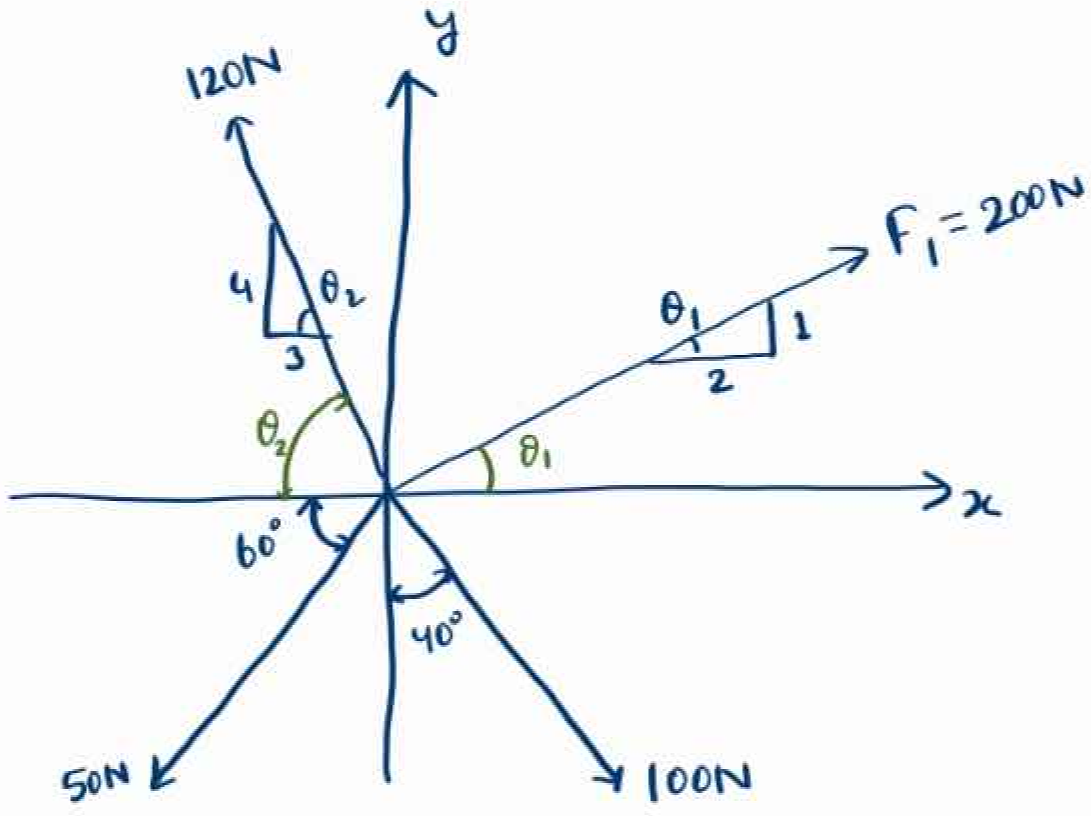
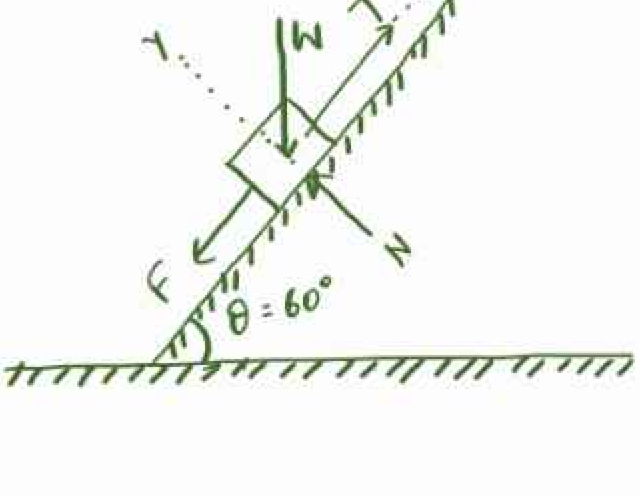
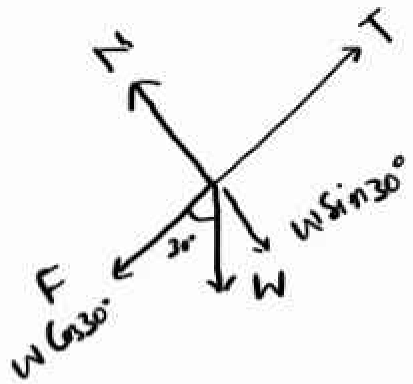


A system of forces acting on a body resting on an inclined plane (fig.). Determine the resultant force if $\theta = 60^\circ$, $W = 1000N$, $N = 500N$, $F = 100N$ and $T = 1200N$



Solⁿ.



$$W = (-1000\cos 30°)i + (-1000\sin 30°)j$$

$N = 500N = 500j$

$F = 100N = -100i$

$T = 1200N = 1200i$

$W = -866.03i - 500j$

$$R = N + F + T + W = \cancel{500}j - 100i + 1200i - 866.03i - \cancel{500}j$$

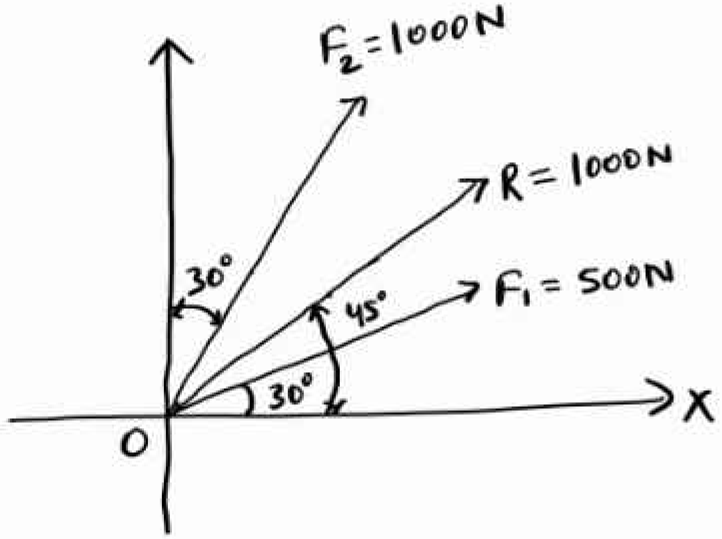
$$= 1200i - 966j$$

$$R = 234j$$

$R = 234N$ ↑ (upward)

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{0}{234} = 0°$$

Two forces acting on a body are 500N and 1000N shown in figure. Determine the third force F_3 such that the resultant of all three forces is 1000N directed at 45° to x-axis as shown in the figure.



Soln Let third force F makes an angle θ with x-axis.

$$R \cos\alpha = \Sigma F_x$$

$$1000 \cos 45^\circ = 500 \cos 30^\circ + 1000 \sin 30^\circ + F \cos\theta$$

$$F \cos\theta = -225.9\,N \quad \text{——— (1)}$$

Similarly

$$R \sin\alpha = \Sigma F_y$$

$$1000 \sin 45^0 = 500 \sin 30^\circ + 1000 \cos 30^\circ + F \sin\theta$$

$$F \sin\theta = -408.9 \quad \text{———} \quad 2$$

Square and add both the equations.

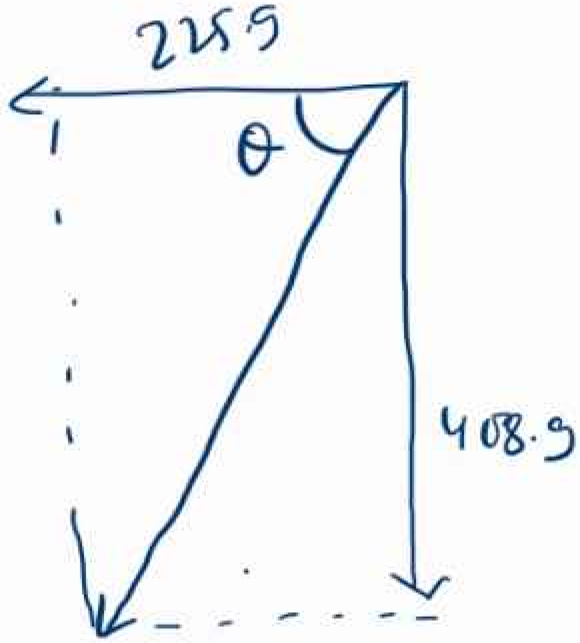
$$F^2 \cos^2\theta + F^2 \sin^2\theta = (-225.9)^2 + (-408.9)^2$$

$$F^2 = (-225.9)^2 + (-408.9)^2$$

$$F = \sqrt{225.9^2 + 408.9^2}$$

$$F = 467.2\text{N}$$

$$\theta = \tan^{-1}\left(\frac{408.9}{225.9}\right) = 61.08^\circ$$


225.9
θ
408.9

Resultant of system of forces

According to Newton's 2^{nd} law of motion of a body moves with uniform acceleration, if it is acted upon by a force. Hence when a system of force act upon a body, the resultant force makes the body to move with uniform acceleration in its direction. If a force equal in magnitude but opposite in sense to the resultant force is applied to the system of forces, the body comes to rest. Such a force which is equal to magnitude but opposite to the resultant force is called equilibrant.

Equilibrant $- E$
Resultant force $= F$

$$E + R = 0$$

Equilibrium of particles subjected to coplanar forces

A body is said to be in equilibrium when, it is at rest or continues to be in steady linear motion. According to Newton's 2^{nd} law of motion, it means the resultant of the force system acting on the body is zero.

Resultant, $R = 0$

$$\Sigma F_x i + \Sigma F_y j = 0$$

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

This is equation of equilibrium for concurrent coplanar system of forces.

Type of forces on a body

Applied forces =

Non-applied forces = Selfweight, Reaction form other bodies.

Self weight = Every object is subjected to gravitational attraction and hence has got self weight

$$W = mg$$

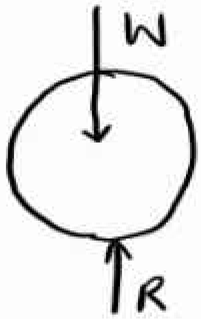
Free Body Diagram

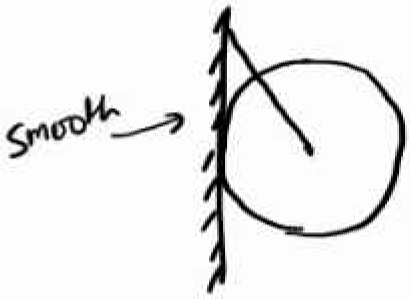
In the analysis, it is necessary to isolate the body under consideration from the other bodies in contact and draw all the forces acting on the body.

First isolated body is drawn and different forces are drawn over it.

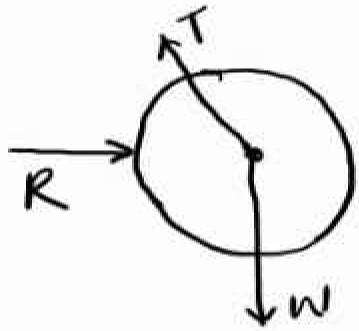
Examples

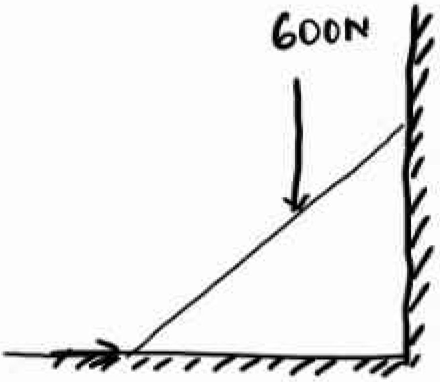
Ball



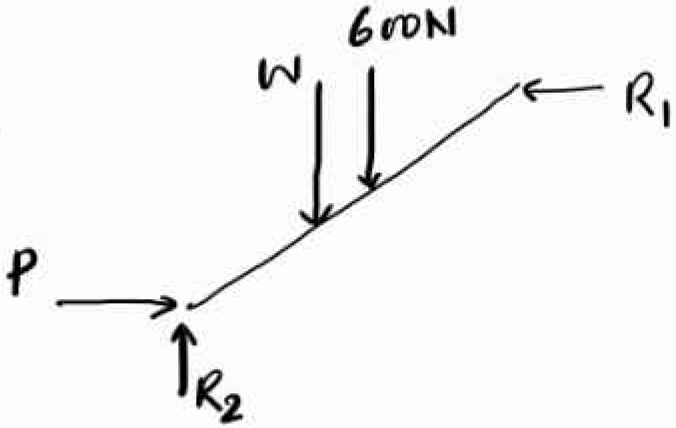


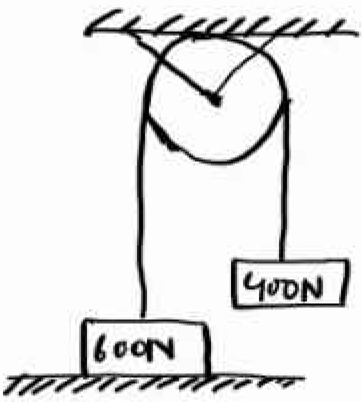
Ball



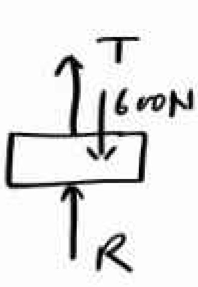


Ladder





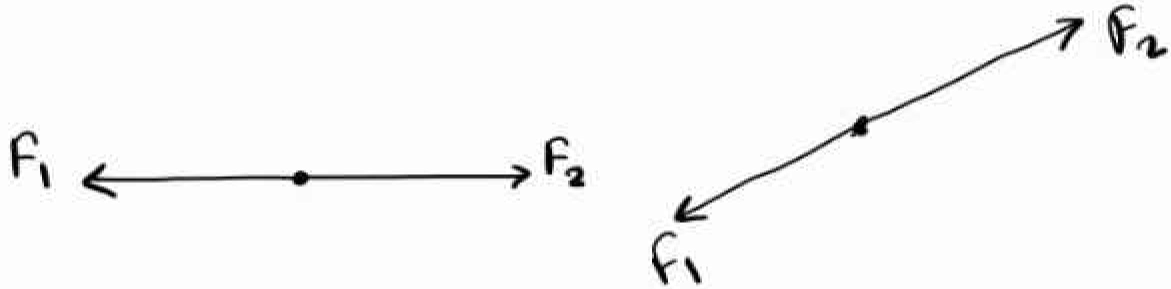
For block weighing 600N



Equilibrium of two force system

When a body is in equilibrium under the action of two forces F_1 & F_2 only, the equilibrium condition is

$$R = 0$$



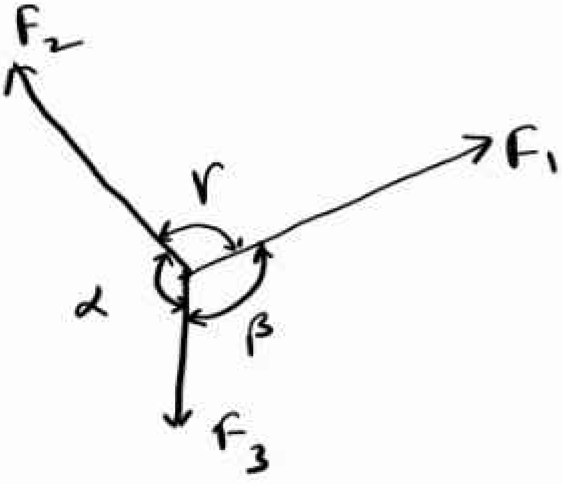
$$F_1 + F_2 = 0$$

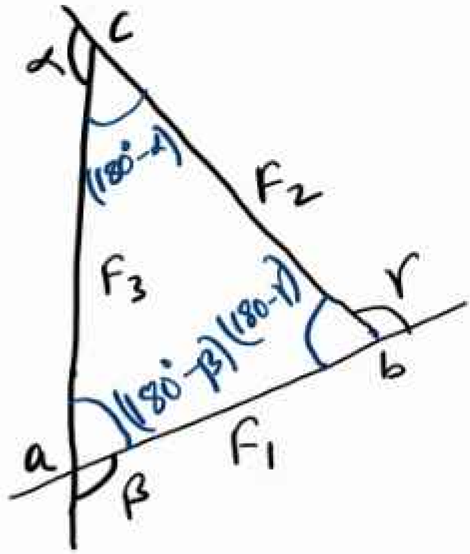
$$F_1 = -F_2$$

Equilibrium of three force system / Lami's theorem

If a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces. This is known as Lami's theorem.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$





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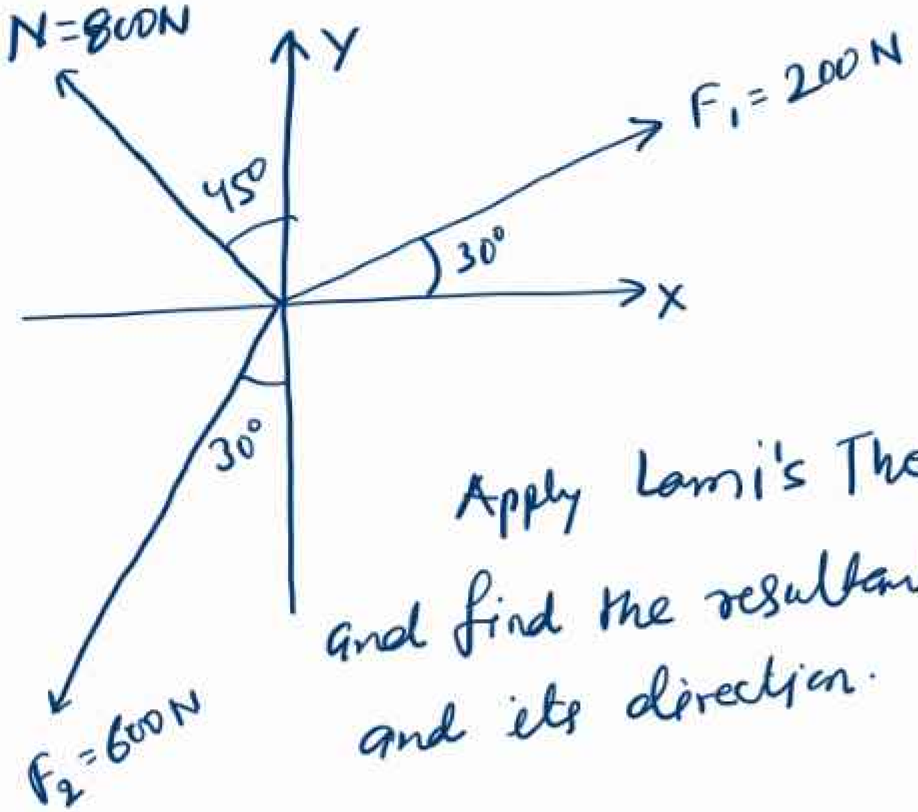
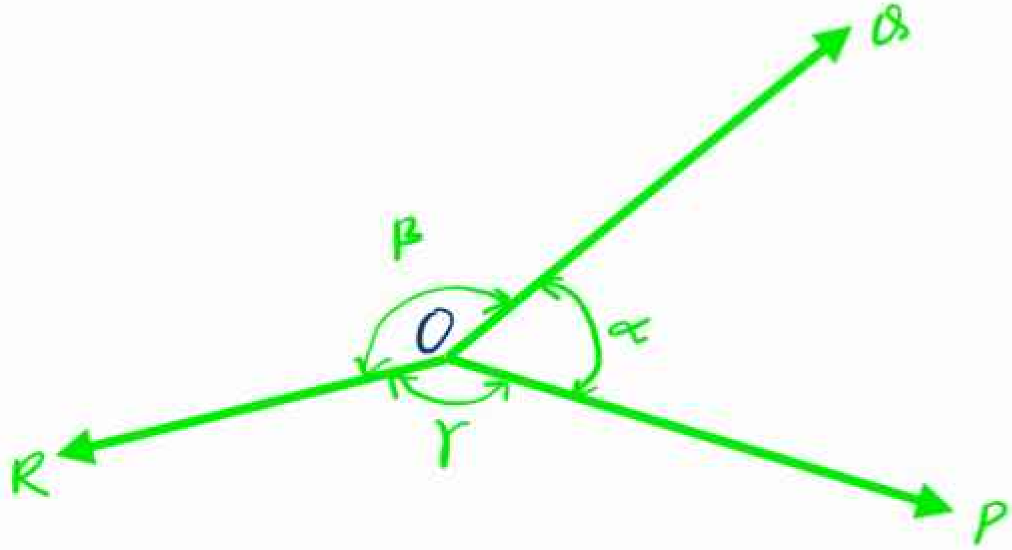


Apply Lami's Theorem and find the resultant force and its direction.

Lami's Theorem

If three forces acting at a point are in equilibrium, each forces will be proportional to the 'sine' of the angle between the other two forces.



Suppose the three forces P, Q, R are acting at a point and they are in equilibrium.

let α = Angle b/w force P & Q
β = Angle b/w force Q & R
and γ = Angle b/w force R & P

$P \propto$ Sine of angle between Q and R

$$\therefore \quad P \propto \sin\beta$$

$$\frac{P}{\sin\beta} = \text{Constant}$$

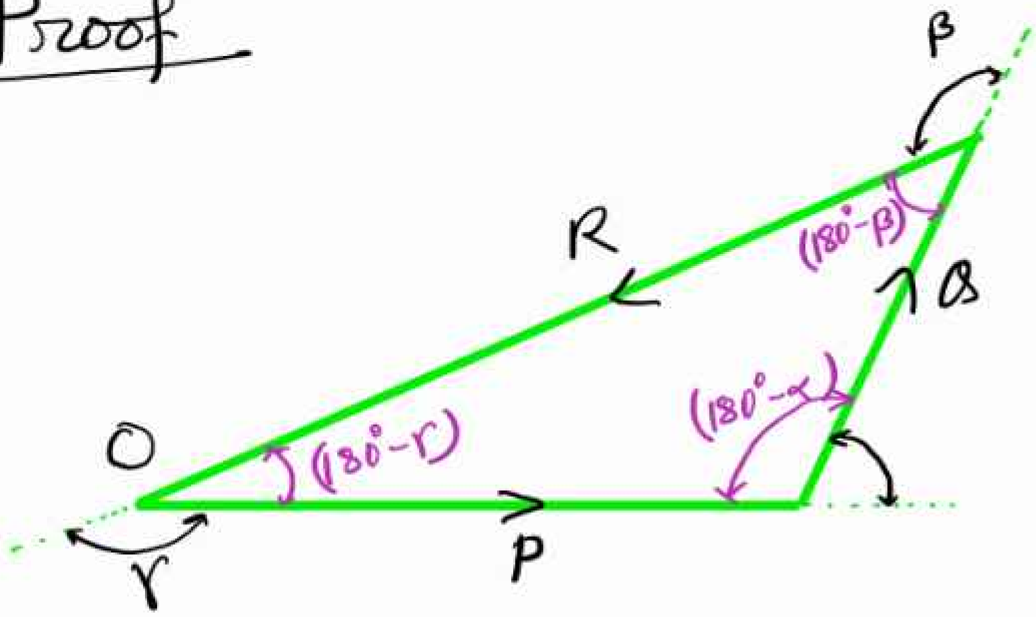
Similarly,

$$\frac{R}{\sin\alpha} = \text{Constant}$$

$$\frac{Q}{\sin\gamma} = \text{Constant}$$

(or) $$\boxed{\frac{P}{\sin\beta} = \frac{R}{\sin\alpha} = \frac{Q}{\sin\gamma}}$$

Proof



Now, apply the sine rule.

$$\frac{P}{\sin(180° - \beta)} = \frac{Q}{\sin(180° - \gamma)} = \frac{R}{\sin(180° - \alpha)}$$

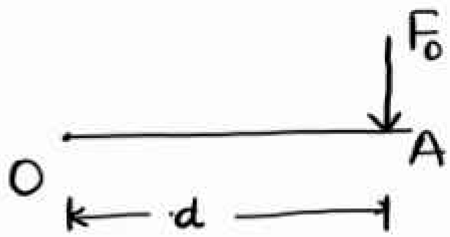
Same expression can be written in the form

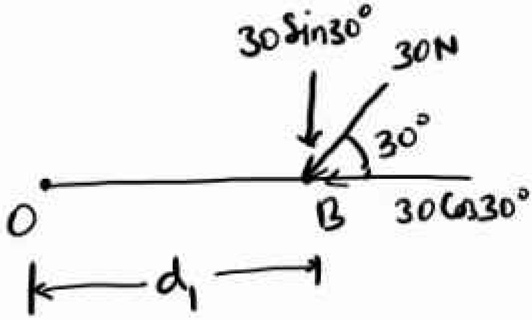
$$\boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}}$$

Moment of force

When a force is applied to a body, it will produce a tendency for the body to rotate about a point that is not on the line of action of the force.

Such tendency of rotation is sometimes referred as torque, but most often it is called the moment of a force (or) simply the moment.



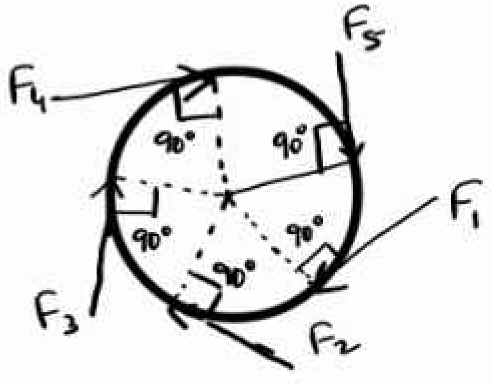


$$M_0 = F_0 d \qquad M_0 = 30 \sin 30^\circ \times d_1$$

Here, d & d_1 = lever arm

F_0 = F_0 is perpendicular load applied on OA

$30 \sin 30^\circ$ = Perpendicular force applied on OB



Let radius of disc = r

$$M = F_0 \cdot r$$

F_0 → Perpendicular force

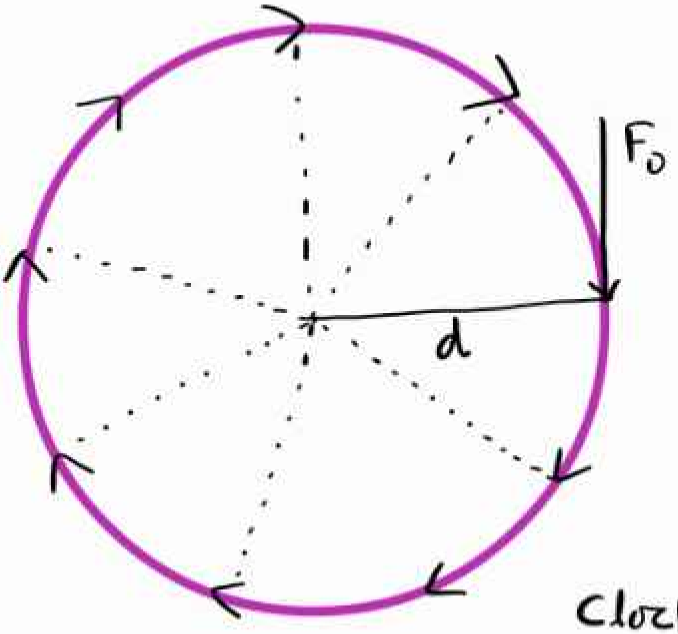
r ← radius of disc

Unit of Force = N

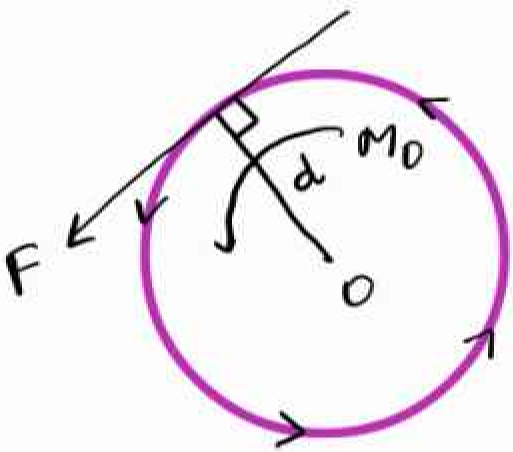
unit of length = m

Unit of moment = N.m

<u>Direction of moment</u>



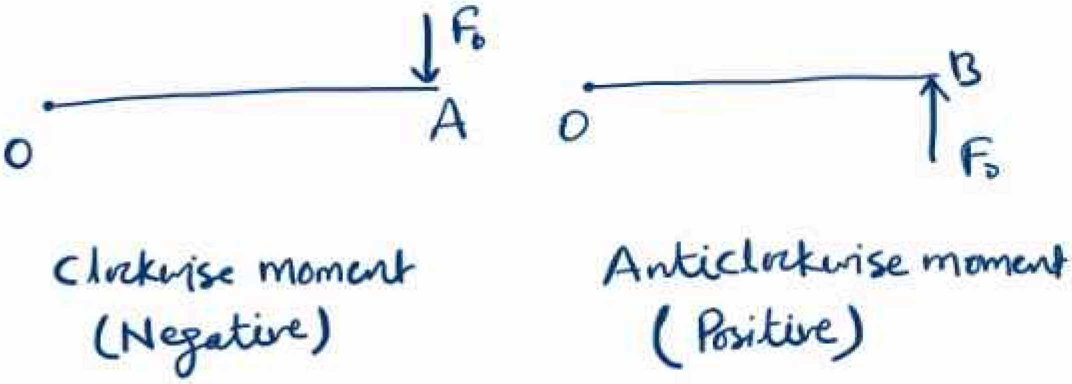
Clock wise rotation

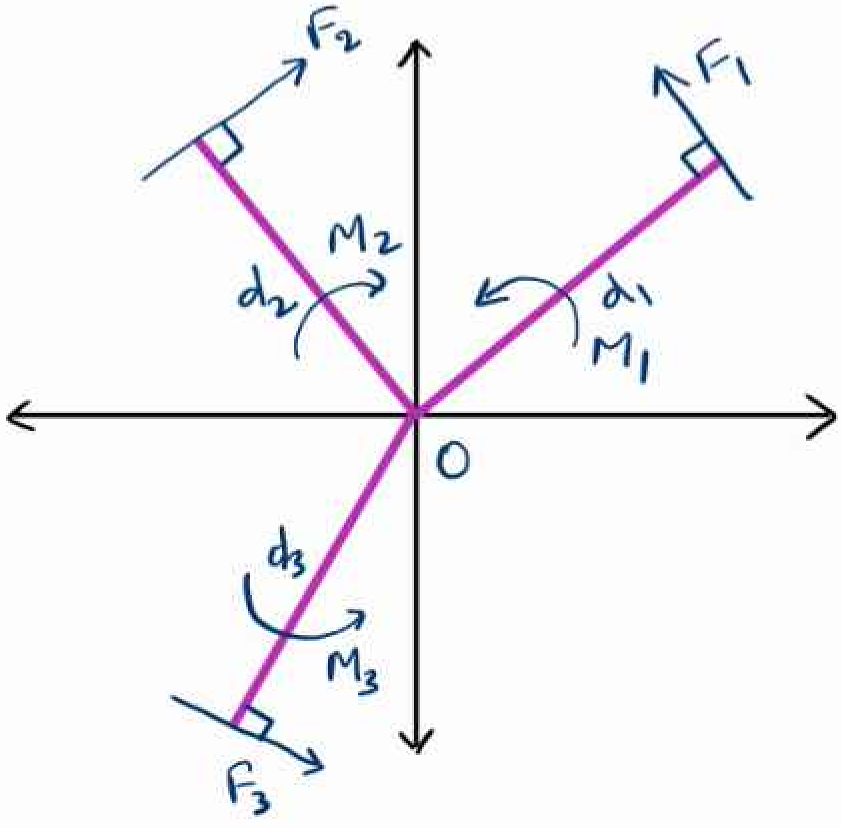


Anticlockwise direction.

Resultant Moment:

For two-dimensional problems, where all the forces lie within the x-y plane, the resultant moment $(M_R)_o$ about point O can be determined by finding the algebraic sum of the moments caused by all the forces in the system. A convension, we will generally consider positive moments as counterclockwise since & clockwise moment will be negative.



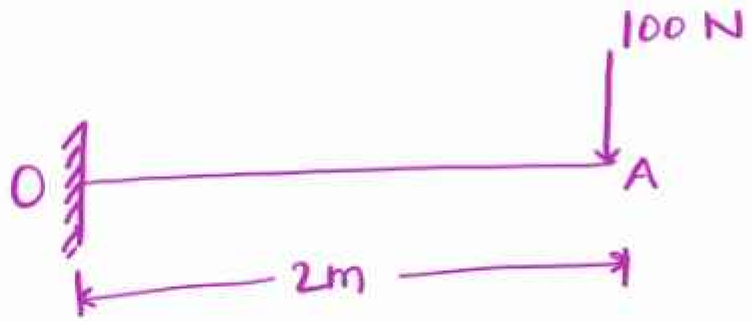


(+ve) (-ve)

$$(M_R)_O = \sum F \cdot d = F_1 d_1 - F_2 d_2 + F_3 d_3$$

Determine the moments

(i)

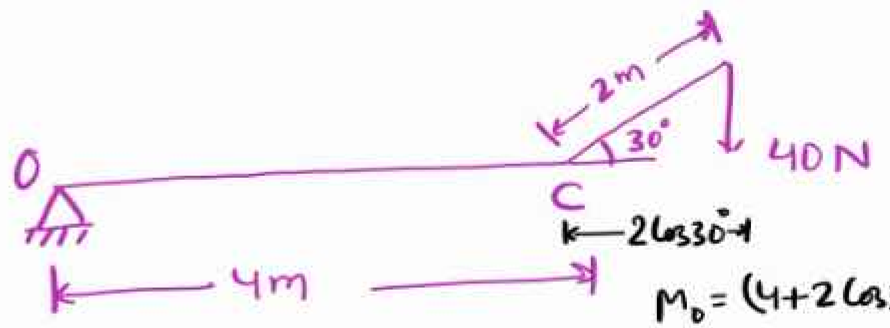


$M_O = 100 \times 2 = 200\,\text{Nm}$

(ii)

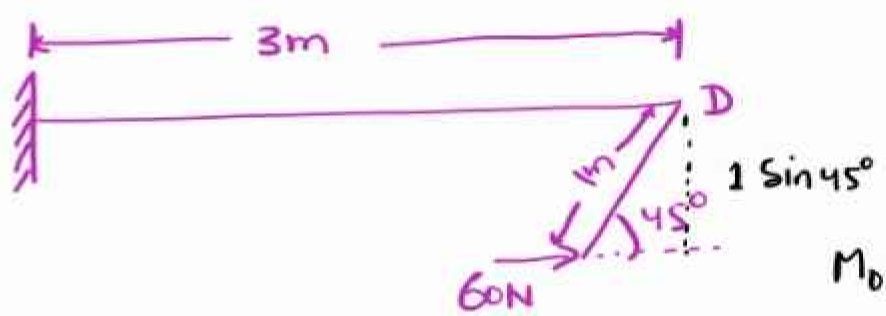
$M_O = 100 \times 0.75\,\text{Nm}$

(iii)



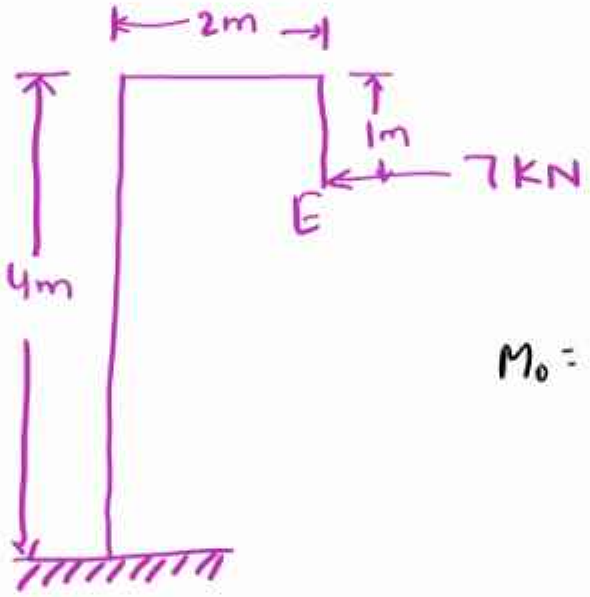
$M_O = (4 + 2\cos 30^\circ) \times 40\,\text{Nm}$

(iv)



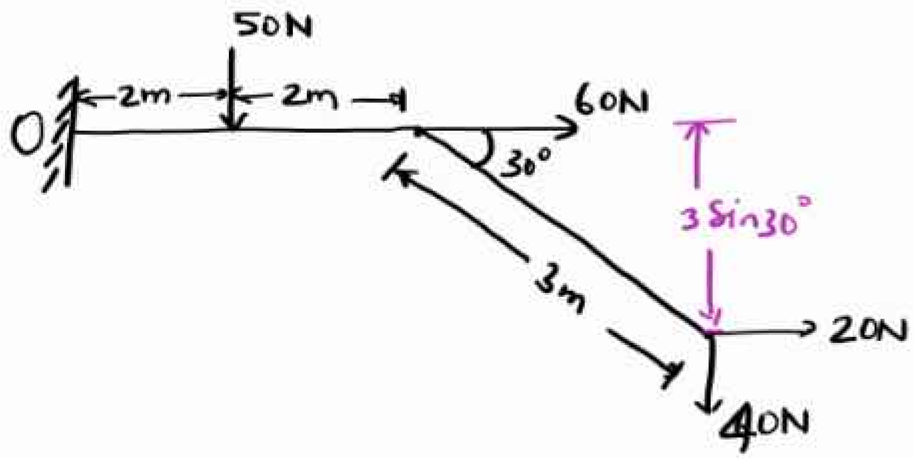
$M_O = 60 \times 1 \sin 45^\circ\,\text{Nm}$

(V)



$M_0 = 7 \times (4-1)\,\mathrm{Nm}$

#



Soln.

$M_1 = -50\times 3 + 60\times 0 - 40\times(4+3\cos 30^\circ) + 20\times 3\sin 30^\circ$

$= -150 + 0 - 40\times 6.598 + 20\times 1.5$

$= -383.92$ N-m

Type of support

① Hinge support

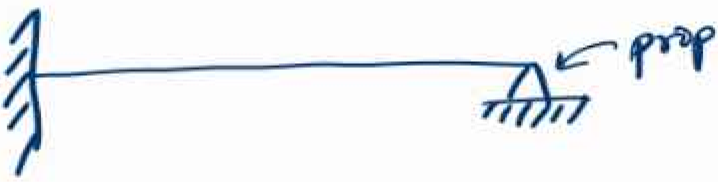
(a)

(b)

(ii) Roller support

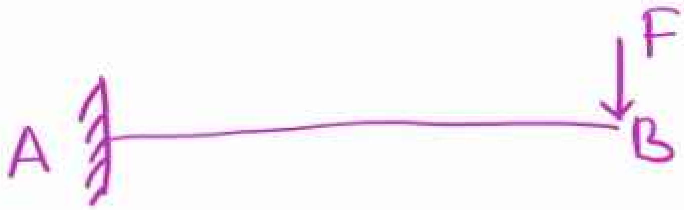
(III) Fixed support

(iv) Propped support

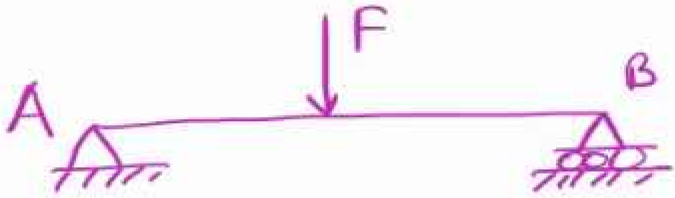


Types of beams

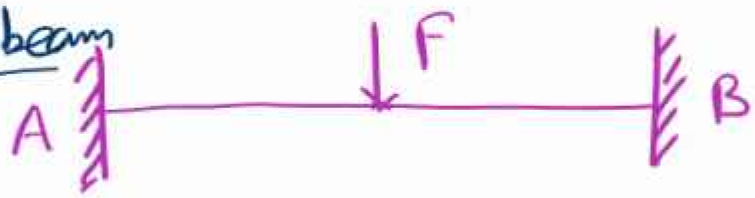
(i) Cantilever beam



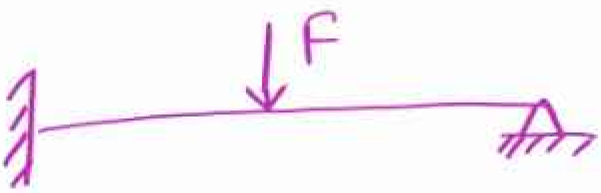
(ii) Simply supported beam



(iii) Fixed beam

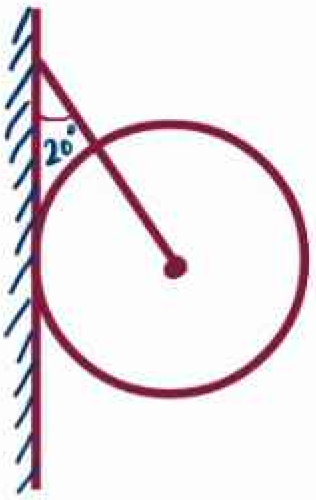


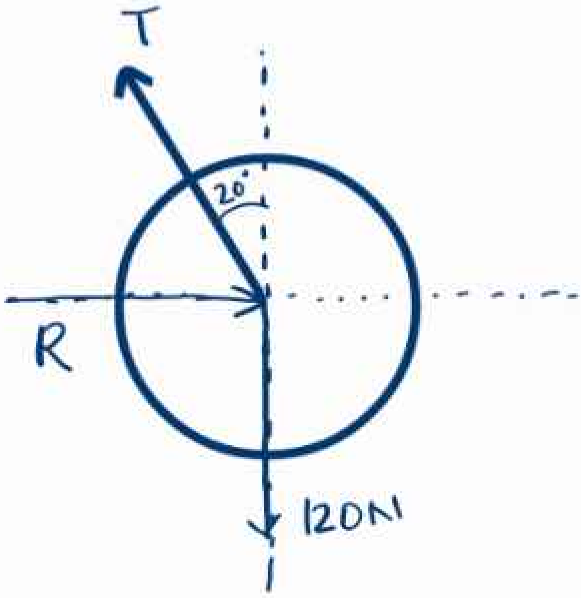
(iv) Propped cantilever beam

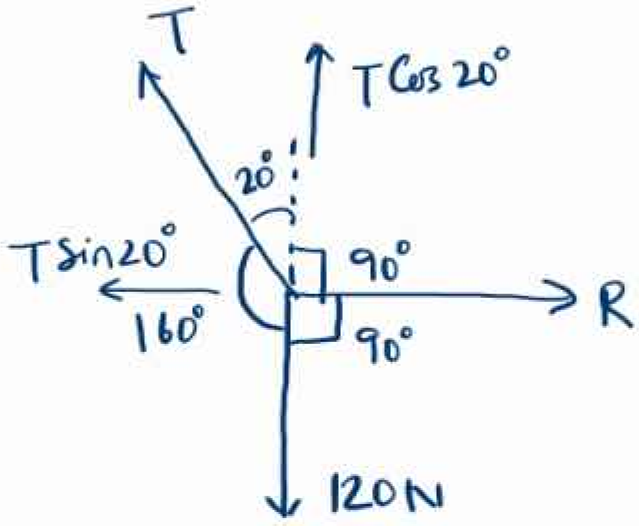


Problem based on Lami's Theorem

A sphere weighting 120N is tied to a smooth wall by a string as shown in figure. Find the tension T in the string and reaction R of the wall.







let's write in form of unit.

$$F_1 = -T\sin 20^\circ i + T\cos 20^\circ j$$

$$F_2 = R\, i$$

$F_3 = -120\,j$

Since, body is under equilibrium

$$F = F_1 + F_2 + F_3 = 0$$

$$= -T\sin 20^\circ\, i + T\cos 20^\circ\, j + R\, i - 120\, j = 0$$

$$\Rightarrow -(T\sin 20^\circ - R)\, i + (T\cos 20^\circ - 120)\, j = 0$$

$$-T\sin 20^\circ + R = 0 \quad \text{——— } ①$$

$$T\cos 20^\circ - 120 = 0 \quad \text{——— } ②$$

from eqn (II)

$$T = \frac{120}{\cos 20^\circ} = 127.7\ \text{N}$$

$$R = 127.7 \times \sin 20^\circ = 43.68\text{N}$$

Alternatively, it can be solved by using Lami's theorem

$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin 160^\circ} = \frac{120}{\sin 110^\circ}$$

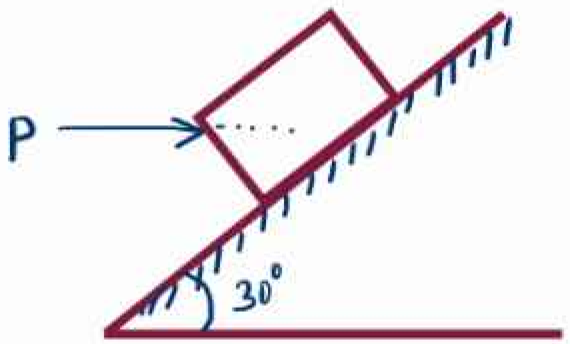
$$\frac{T}{\sin 90^\circ} = \frac{120}{\sin 110^\circ}$$

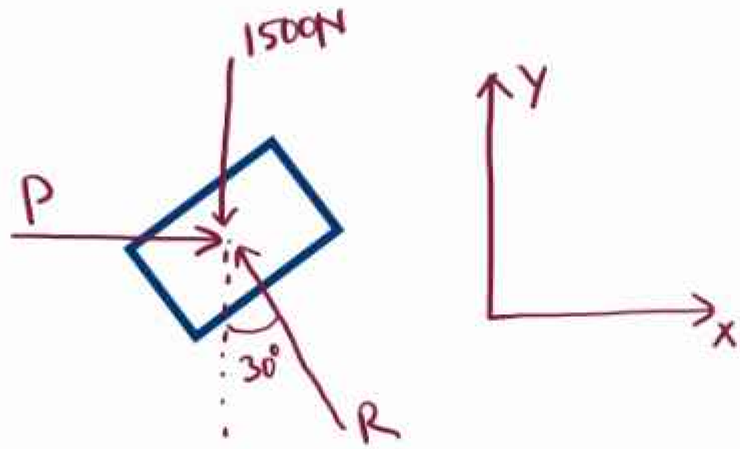
$$T = \frac{120 \times \sin 90^\circ}{\sin 110^\circ} = 127.7 \text{ N}$$

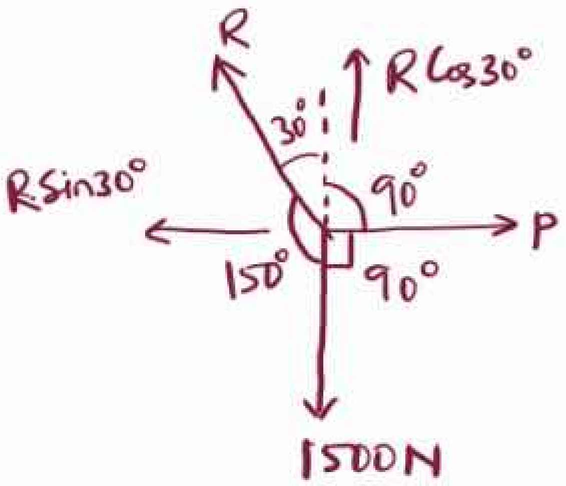
$$\frac{R}{\sin 160^\circ} = \frac{120}{\sin 110^\circ}$$

$$R = 120 \times \frac{\sin 160^\circ}{\sin 110^\circ} = 43.68 \text{ N}$$

Determine the horizontal force P to be applied to a block weighting 1500 N to hold it in the position. The inclined plane is smooth and makes 30° with the horizontal.







$$F_1 + F_2 + F_3 = 0$$

$$F_1 = R\cos 30^\circ j - R\sin 30^\circ i$$

$$F_2 = P i$$

$$F_3 = -1500 j$$

$$R\cos 30^\circ j - R\sin 30^\circ i + P i - 1500 j = 0$$

$$(P - R\sin 30^\circ) i + (R\cos 30^\circ - 1500) j = 0$$

$$P - R\sin 30^\circ = 0 \quad \text{——— (I)}$$

$$R\cos 30^\circ - 1500 = 0 \quad \text{——— (II)}$$

from eq^n (II)

$$R\cos 30^\circ - 1500 = 0$$

$$R = \frac{1500}{\cos 30^\circ} = 1732N$$

$$P = 1732 \times \frac{1}{2} = 866\,N$$

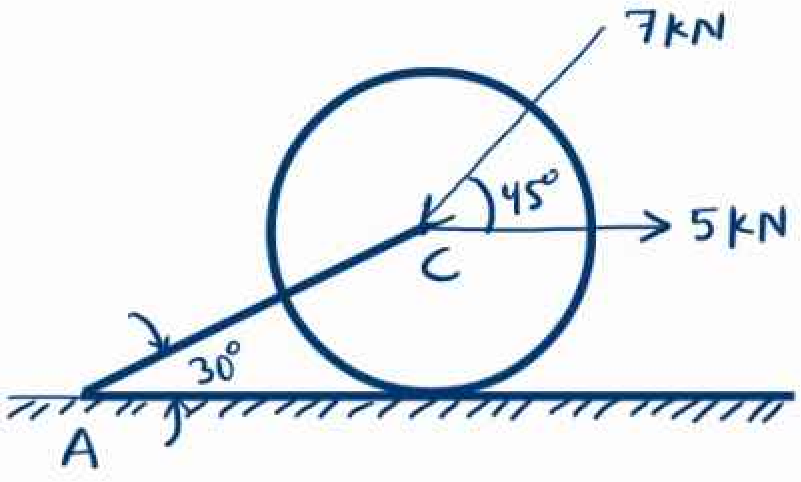
Alternatively, using Lami's theorem.

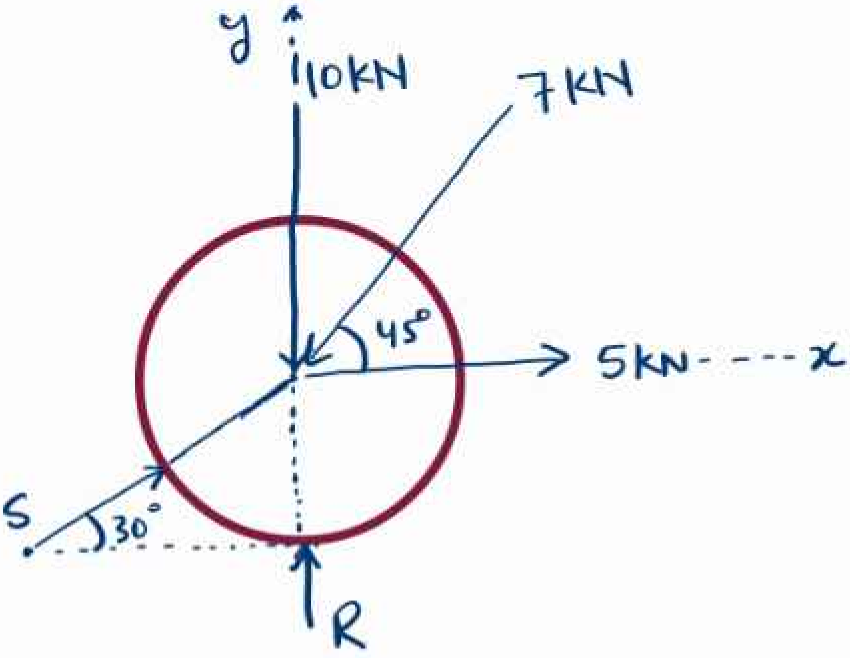
$$\frac{P}{\sin 150^\circ} = \frac{R}{\sin 90^\circ} = \frac{1500}{\sin 120^\circ}$$

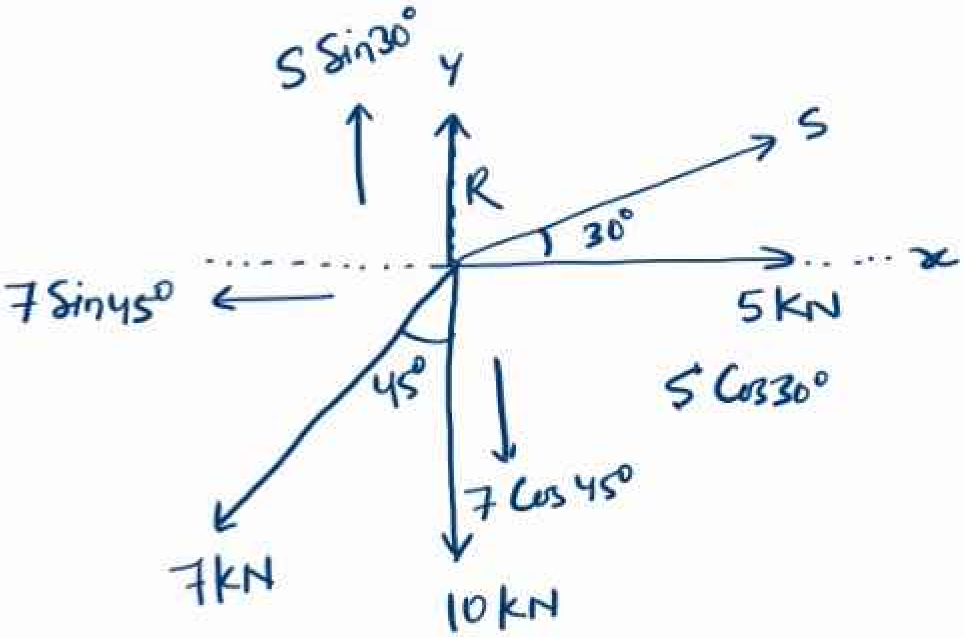
$$P = 1500 \times \frac{\sin 150^\circ}{\sin 120^\circ} = 866\,N$$

$$R = 1500 \times \frac{\sin 90^\circ}{\sin 120^\circ} = 1732\,N$$

A roller weighting 10 kN rests on a smooth horizontal floor. It is connected to the floor using bar AC. Determine the force in the bar AC and reaction from the floor, if roller is subjected to a horizontal force of 5 kN and an inclined force of 7 kN as shown in figure.







As object is in equilibrium, so. sum of all the forces will be zero.

$$F_1 + F_2 + F_3 + F_4 + F_5 = 0 \quad \text{—— (1)}$$

$F_1 = 5i$

$F_2 = S\cos 30^\circ i + S\sin 30^\circ j$

$F_3 = -7\sin 45^\circ i - 7\cos 45^\circ j$

$F_4 = -10j$

$F_5 = Rj$

From eq^n ①

$5i + 5\cos 30^\circ i + 5\sin 30^\circ j - 7\sin 45^\circ i - 7\cos 45^\circ j - 10j = 0$

$(5 + 5\cos 30^\circ - 7\sin 45^\circ) i + (5\sin 30^\circ - 7\cos 45^\circ - 10 + R) j = 0$

All the forces in x & y direction will be equal to zero individually.

$(5 + 5\cos 30^\circ - 7\sin 45^\circ) = 0$ ——— (ii)

$(5\sin 30^\circ - 7\cos 45^\circ - 10 + R) = 0$ ——— (iii)

from eq^n ①

$5\sin 30^\circ = 7\sin 45^\circ - 5$

$5 = \dfrac{7\sin 45^\circ - 5}{\sin 30^\circ} = \dfrac{7 \times \frac{1}{\sqrt{2}} - 5}{\frac{1}{2}}$

$5 = -0.1005\ N$

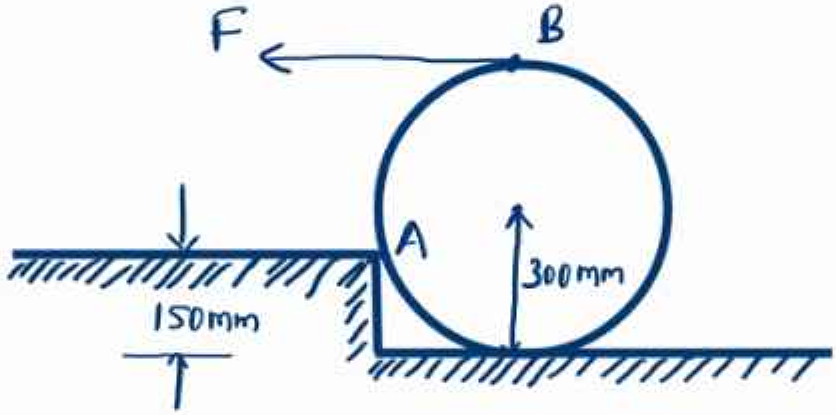
$R = 10 + 7\cos 45^\circ - 5 \times \sin 30^\circ$

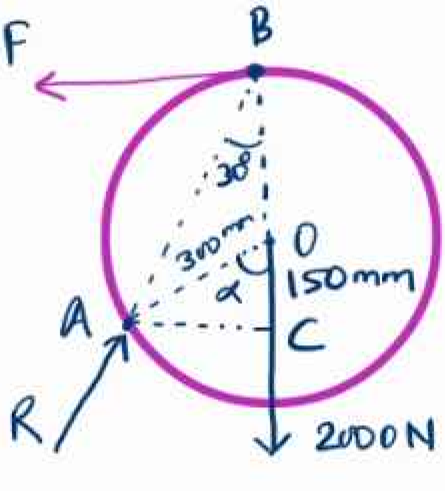
$$R = 10 + \frac{7}{\sqrt{2}} + 0.1005 \times \frac{1}{2}$$

$$= 14.949 + 0.05025$$

$$R = 14.999\ N$$

A roller of radius $r = 300mm$, weighting 2000N is to be pulled over a curb of height 150mm by horizontal force F applied to the end of a string wound tightly around the circumference of the roller. Find the magnitude of F required to start the roller move over the curb. What is the least F through centre of roller to just turn it over the curve?





$$\cos\alpha = \frac{150}{300} = \frac{1}{2}$$

$$\alpha = 60^\circ$$

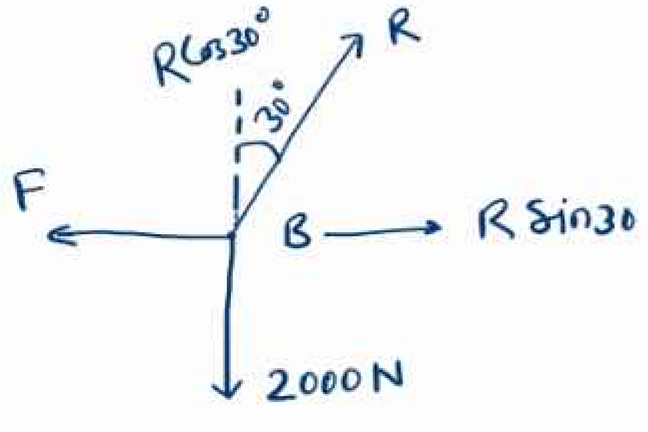
In ΔAOB

$\angle OAB = \angle OBA$, since $OA = OB = r$

$$\angle OAB + \angle OBA = \alpha = 60^\circ$$

$$2\angle OBA = 60^\circ$$

$$\angle OBA = 30^\circ$$



By considering equilibrium condition, resultant of forces will be equal to zero.

$$F_1 + F_2 + F_3 = 0$$

$$F_1 = -2000\,j$$

$$F_2 = -F\,i$$

$$F_3 = R\sin 30^\circ i + R\cos 30^\circ j$$

$$-2000j - Fi + R\sin 30^\circ i + R\cos 30^\circ j = 0$$

$$(R\sin 30^\circ - F)\,i + (R\cos 30^\circ - 2000)\,j = 0$$

$$R\sin 30^\circ - F = 0 \quad \text{——— (I)}$$

$$R\cos 30^\circ - 2000 = 0 \quad \text{——— (II)}$$

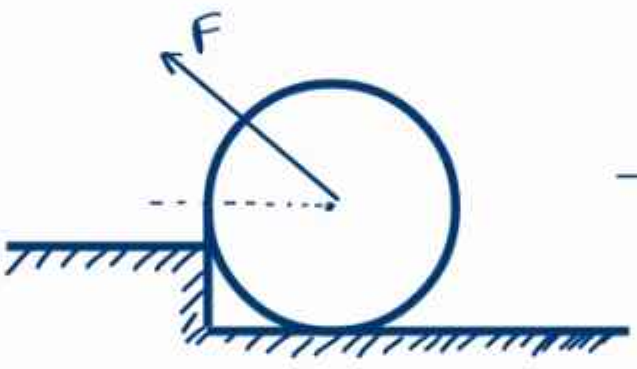
From eq^n (II)

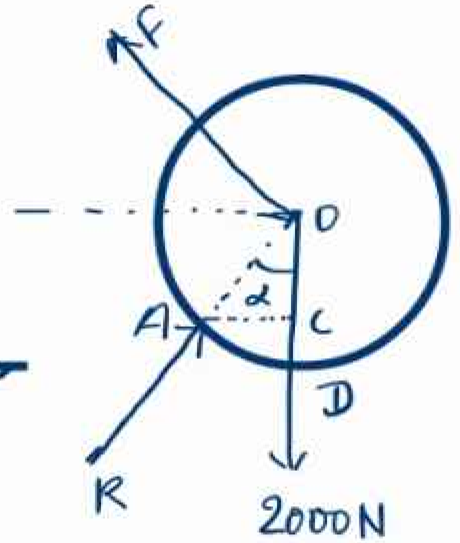
$$R = \frac{2000}{\cos 30^\circ} = \frac{2000 \times 2}{\sqrt{3}} = 2309.4\ \text{N}$$

$$F = R\sin 30^\circ$$

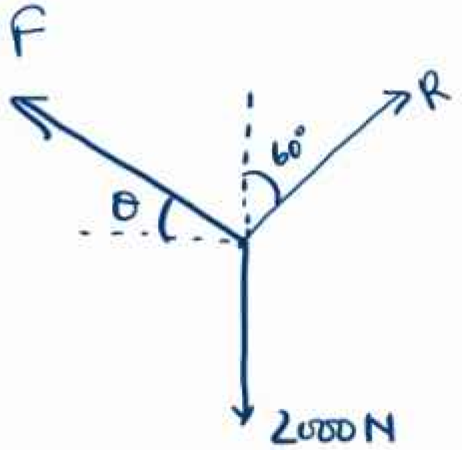
$$\boxed{F = 2309.4 \times \frac{1}{2} = 1154.7\ \text{N}}$$

least force from centre of roller





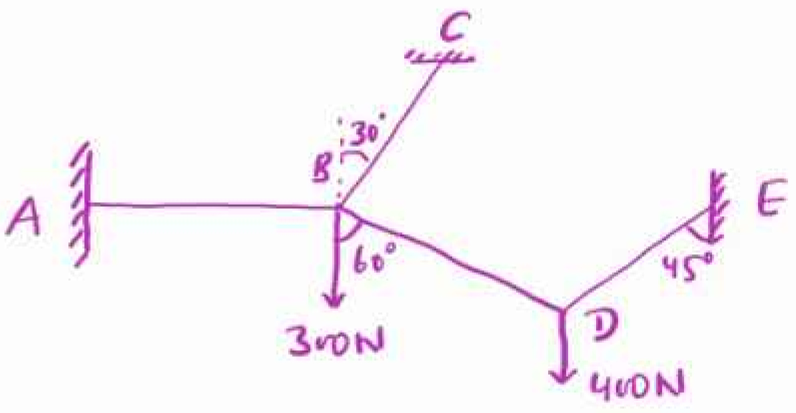
$\alpha = 60^\circ$

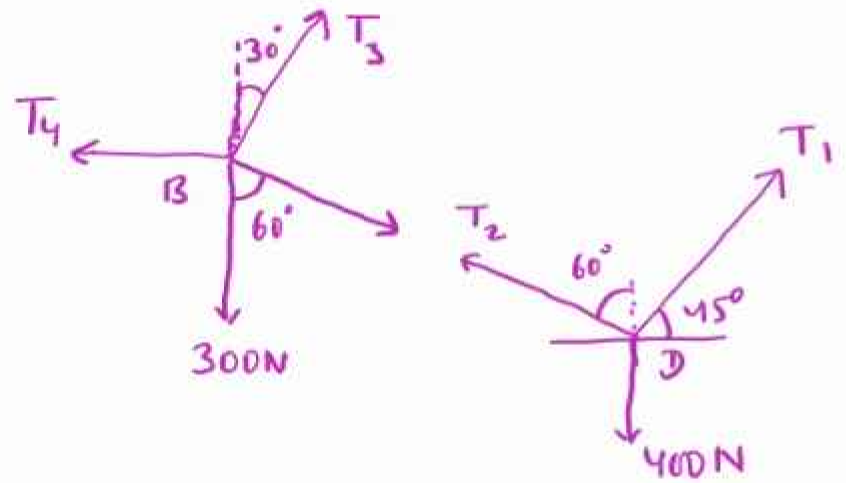


Equilibrium of connected bodies

When two (or) more bodies are in contact with each other, the system of forces appears to be non-concurrent force system. However, when each body considered saperately, in many situation it terms out to be a set of concurrent force system.

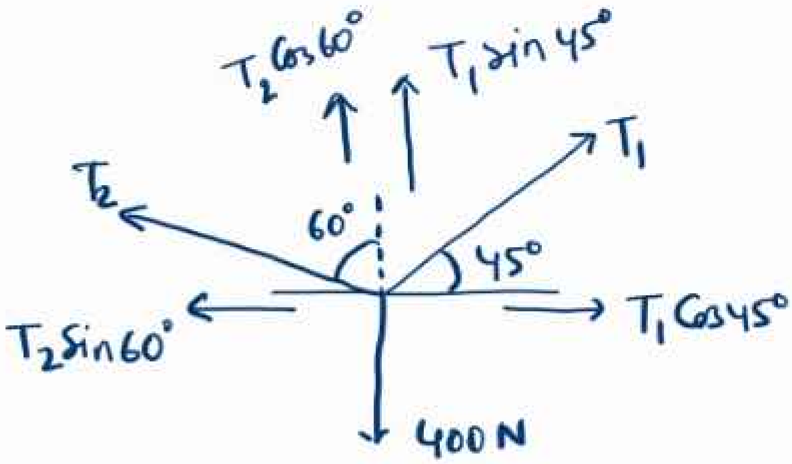
\# A system of connected flexible cables shown in figure. is supporting two vertical forces 300N and 400N at point B & D. Determine the forces in various segment of the cable.





Soln.

From equilibrium condition of point D.



$$T_1 \cos 45^\circ i + T_1 \sin 45^\circ j + T_2 \cos 60^\circ j - T_2 \sin 60^\circ i - 400 j = 0$$

$$(T_1 \cos 45^\circ - T_2 \sin 60^\circ) i + (T_1 \sin 45^\circ + T_2 \cos 60^\circ - 400) j = 0$$

$$T_1 \cos 45^\circ - T_2 \sin 60^\circ = 0 \qquad (1)$$

$$T_1 = T_2 \frac{\sin 60^\circ}{\cos 45^\circ} = 1.225 T_2$$

$$T_1 \sin 45^\circ + T_2 \cos 60^\circ - 400 = 0 \qquad (2)$$

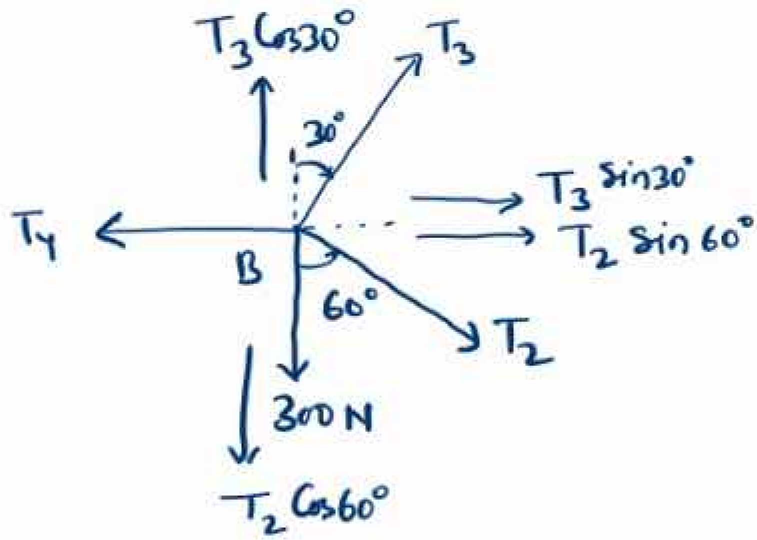
$$1.225 \times \sin 45^\circ \times T_2 + T_2 \cos 60^\circ = 400$$

$$T_2 = \frac{400}{(1.225 \sin 45^\circ + \cos 60^\circ)}$$

$$T_2 = 292.78 \text{ N}$$

$$T_1 = 1.225 \times 292.78 = 358.66 \text{ N}$$

Now, considering the equilibrium of point B.



$$T_2 \sin 60^\circ i + T_3 \sin 30^\circ i - T_4 i + T_3 \cos 30^\circ j - T_2 \cos 60 j - 300 j = 0$$

$$(T_2 \sin 60^\circ + T_3 \sin 30^\circ - T_4) i + (T_3 \cos 30^\circ - T_2 \cos 60^\circ - 300) j = 0$$

$$T_2 \sin 60^\circ + T_3 \sin 30^\circ - T_4 = 0 \quad \text{——— (3)}$$

$$T_3 \cos 30^\circ - T_2 \cos 60^\circ - 300 = 0 \quad \text{——— (4)}$$

From (4)

$$T_3 \cos 30^\circ - T_2 \cos 60^\circ = 300$$

$$T_3 \cos 30^\circ = 300 + 292.78 \times \cos 60^\circ$$

$$T_3 = \frac{300 + 292.78 \cos 60^\circ}{\cos 30^\circ}$$

$$T_3 = 515.45 \text{ N}$$

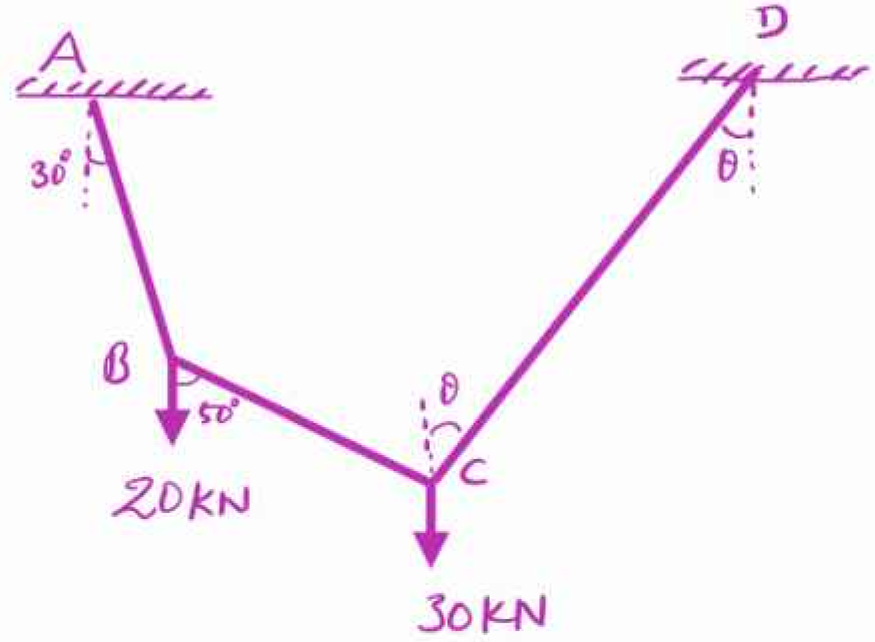
From eq^n (3)

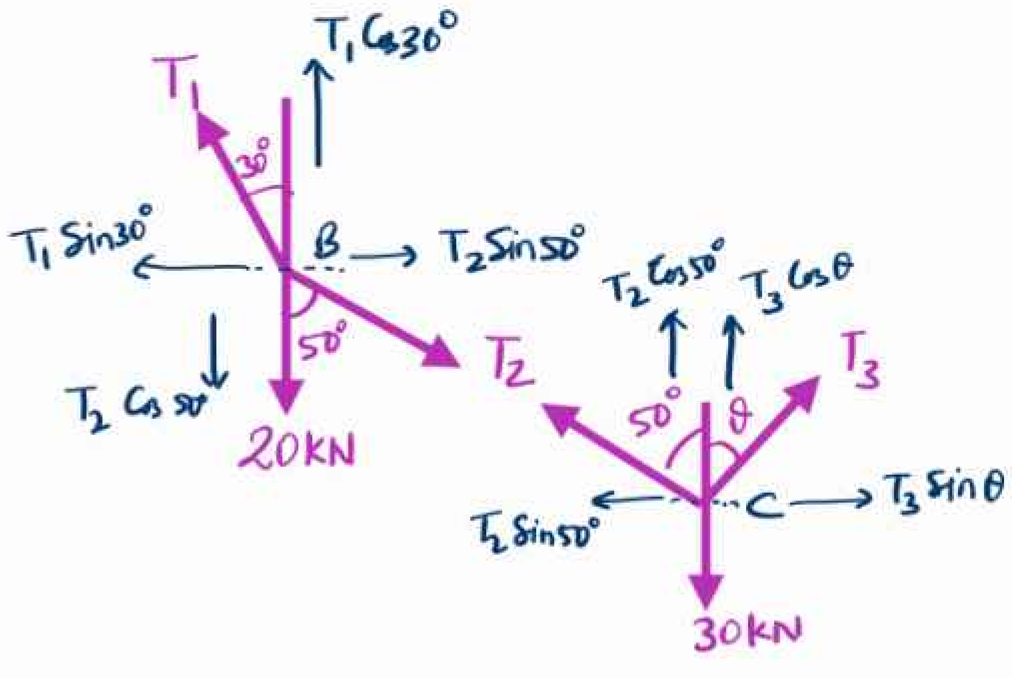
$$T_2 \sin 60^\circ + T_3 \sin 30^\circ - T_4 = 0$$

$$292.78 \sin 60^\circ + 515.45 \sin 30^\circ = T_4$$

$$T_4 = 511.28 \text{ N}$$

A wire rope is fixed at two points A & D as shown in fig. Two weights 20 kN and 30 kN are suspended from B & C respectively. The inclination of chords AB and BC are at 30° & 50° respectively to the vertical. Find the forces in segments AB, BC & CD. Determine the inclination of the segment CD to vertical.





Solⁿ. From the equilibrium condition of point B.

$$-T_1 \sin 30^\circ i + T_2 \sin 50^\circ i + T_1 \cos 30^\circ j - T_2 \cos 50^\circ j - 20 j = 0$$

$$(-T_1 \sin 30^\circ + T_2 \sin 50^\circ) i + (T_1 \cos 30^\circ - T_2 \cos 50^\circ - 20) j = 0$$

$$-T_1 \sin 30^\circ + T_2 \sin 50^\circ = 0 \quad \text{——— (1)}$$

$$T_1 \cos 30^\circ - T_2 \cos 50^\circ - 20 = 0 \quad \text{——— (2)}$$

Solve for T_1 & T_2

From the equilibrium condition of point C

$$T_3 \sin\theta \, i - T_2 \sin 50^\circ i + T_2 \cos 50^\circ j + T_3 \cos\theta \, j - 30 j = 0$$

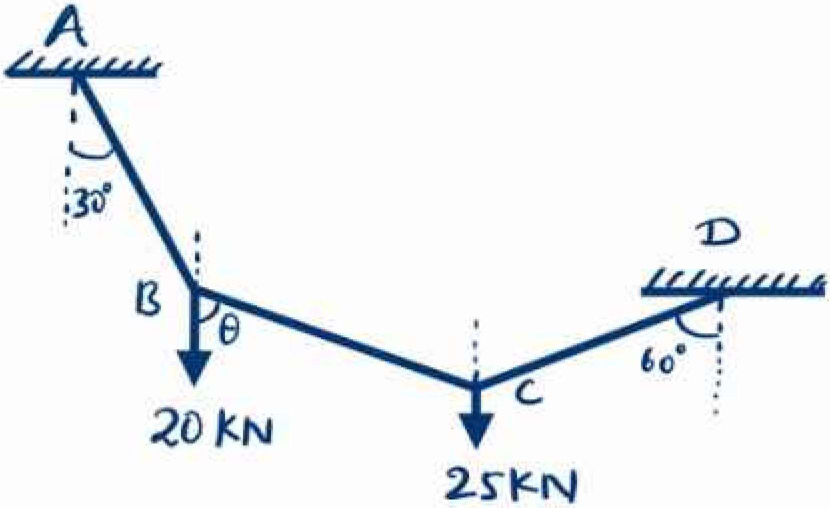
$$(T_3 \sin\theta - T_2 \sin 50^\circ) i + (T_2 \cos 50^\circ + T_3 \cos\theta - 30) j = 0$$

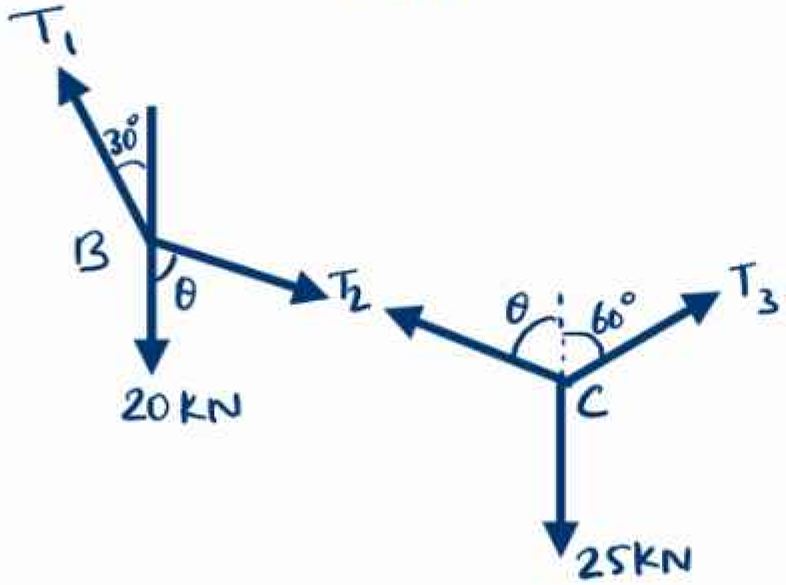
$$T_3 \sin\theta - T_2 \sin 50^\circ = 0 \quad \text{——— (3)}$$

$$T_2 \cos 50^\circ + T_3 \cos\theta - 30 = 0 \quad \text{——— (4)}$$

Solve for T_3 & θ

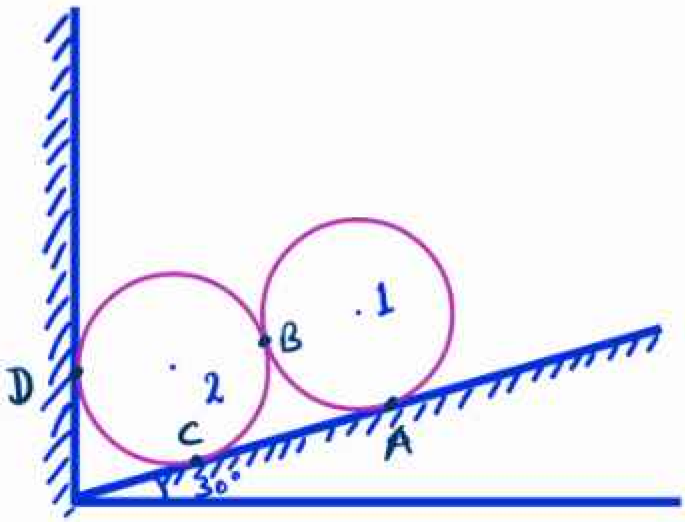
A wire is fixed at two points A & D as shown in figure. Two weights 20 KN & 25 KN are suspended at B & C respectively. When equilibrium is reached it is found that inclination of AB is 30° and that of CD is 60° to the vertical. Determine the tension in the segment AB, BC and CD of the rope and also the inclination of BC to the vertical.

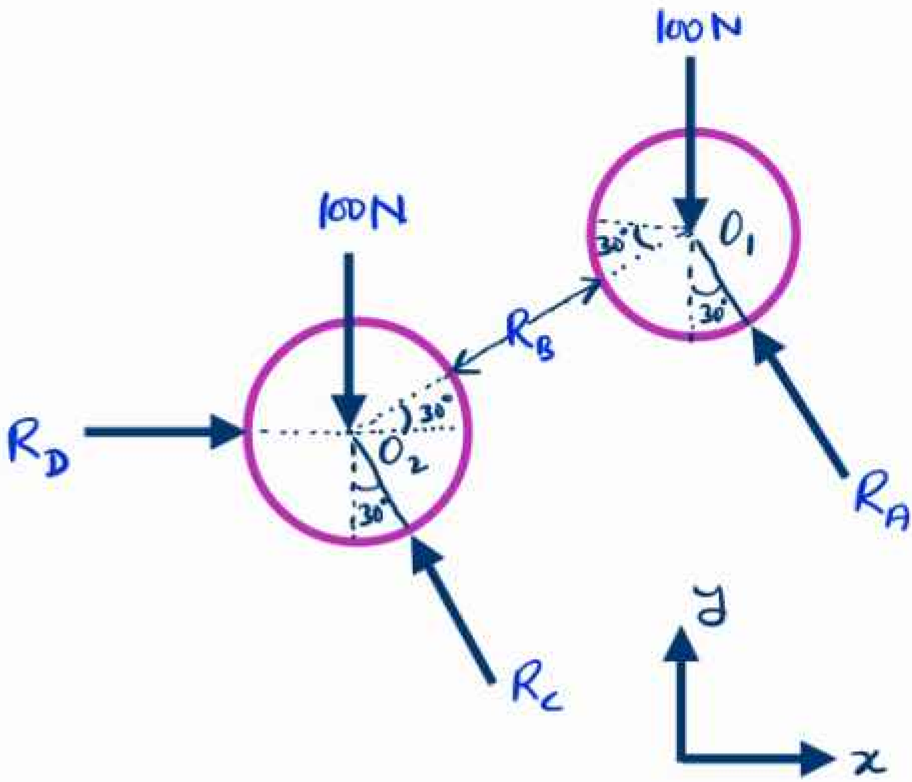


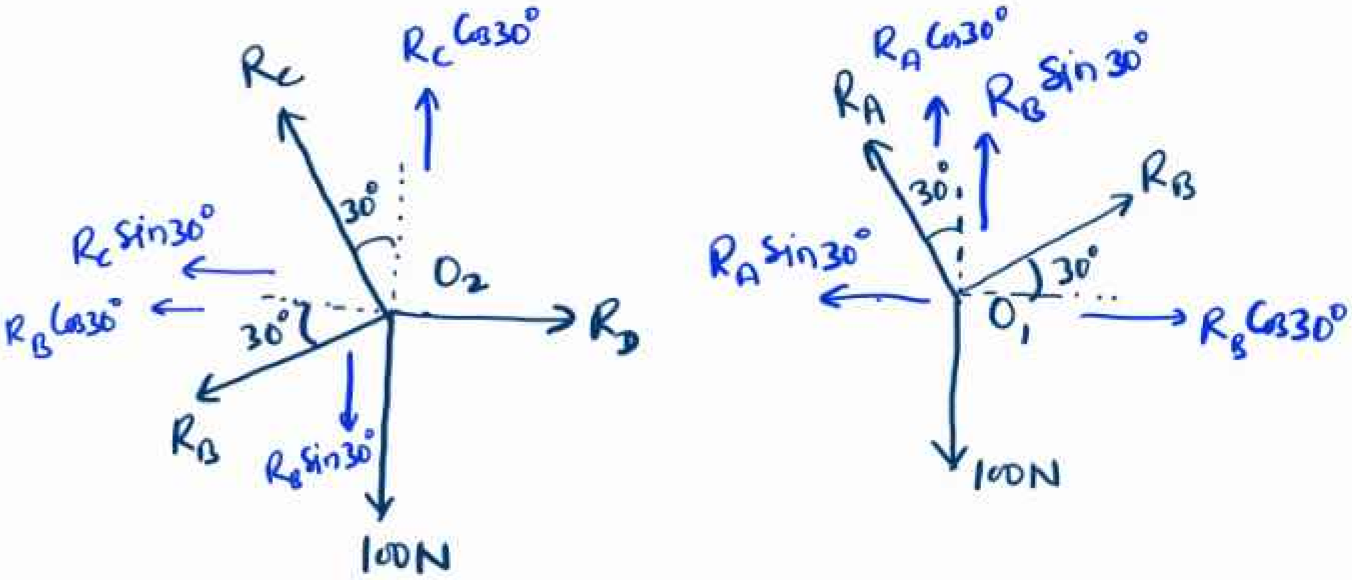


Please practic this problem:-

Two identical rollers, each weighting 100 N are supported by an inclined plane and a vertical wall as shown in figure. Assuming all contact surface are smooth, find the reaction developed at the contact surfaces A, B, C & D.







<u>For equilibrium condition of ball 1</u>

$$R_B \cos 30^\circ i - R_A \sin 30^\circ i + R_B \sin 30^\circ j + R_A \cos 30^\circ j - 100 j = 0$$

$$(R_B \cos 30^\circ - R_A \sin 30^\circ) i + (R_B \sin 30^\circ + R_A \cos 30^\circ - 100) j = 0$$

$$R_B \cos 30^\circ - R_A \sin 30^\circ = 0 \quad \text{——— (1)}$$

$$R_B \sin 30^\circ + R_A \cos 30^\circ - 100 = 0 \quad \text{——— (2)}$$

$$R_B = R_A \cdot \frac{\sin 30^\circ}{\cos 30^\circ} = 0.58 R_A$$

$$0.58 R_A \times \sin 30^\circ + R_A \cos 30^\circ = 100$$

$$R_A = \frac{100}{0.58 \times \sin 30^\circ + \cos 30^\circ} = 86.5\,N$$

$$R_B = 0.58 \times 86.5 = 50.2\,N$$

For equilibrium condition of ball 2

$$R_D i - R_C \sin 30^\circ i - R_B \cos 30^\circ i + R_C \cos 30^\circ j - R_B \sin 30^\circ j - 100 j = 0$$

$$(R_D - R_C \sin 30^\circ - R_B \cos 30^\circ) i + (R_C \cos 30^\circ - R_B \sin 30^\circ - 100) j = 0$$

$$R_D - R_C \sin 30^\circ - R_B \cos 30^\circ = 0 \quad \text{——— (3)}$$

$$R_C \cos 30^\circ - R_B \sin 30^\circ - 100 = 0 \quad \text{——— (4)}$$

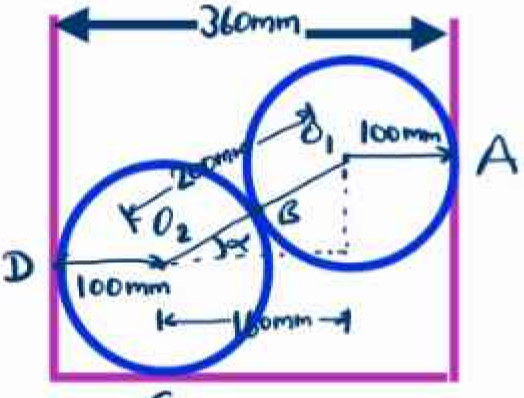
$$R_C \cos 30^\circ - 50.2 \times \sin 30^\circ - 100 = 0$$

$$R_C = \frac{50.2 \sin 30^\circ + 100}{\cos 30^\circ} = 144.45 \text{ N}$$

$$R_D = 144.45 \sin 30^\circ + 50.2 \times \cos 30^\circ$$

$$R_D = 115.7 \text{ N}$$

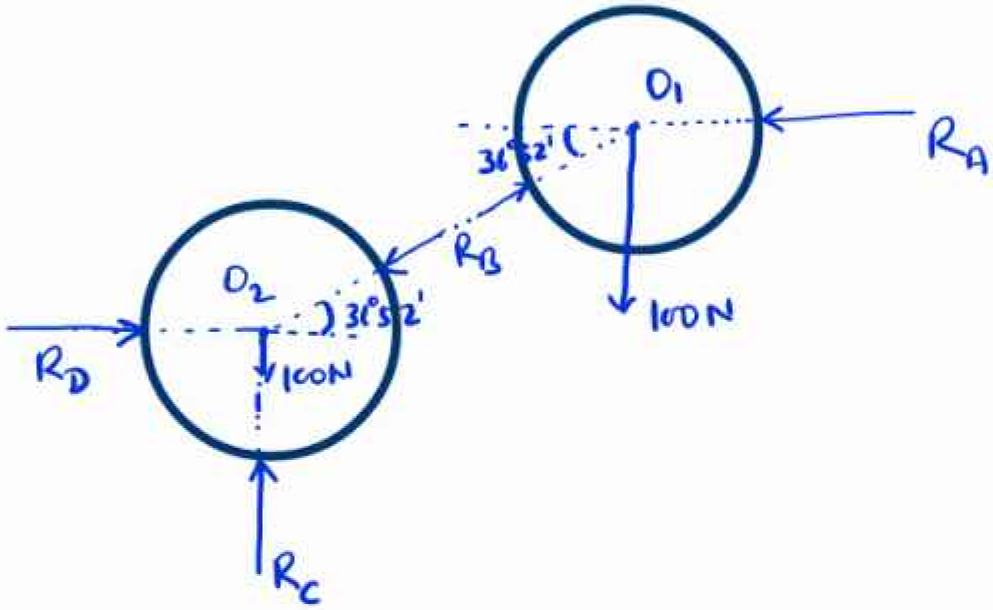
Two smooth spheres each of radius 100mm, and weight 100 N, rest in a horizontal channel having vertical walls, the distance between the walls being 360mm. Find the reaction at the points of contacts A, B, C & D.

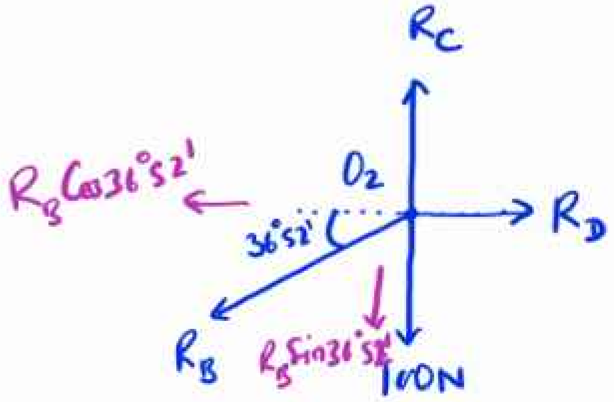


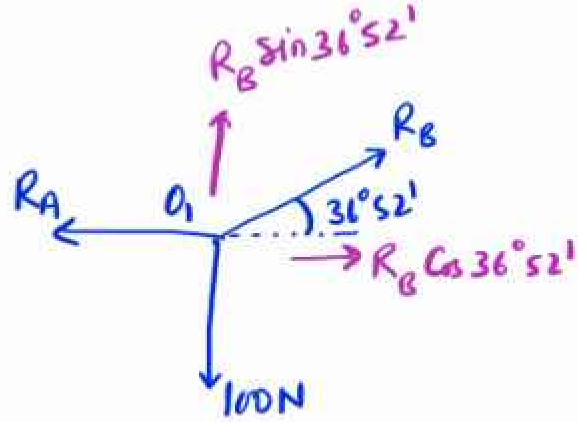
$$\cos\alpha = \frac{160}{200}$$

$$\alpha = \cos^{-1}\left(\frac{160}{200}\right)$$

$$\alpha = 36^\circ 52'$$







For equilibrium condition of sphere ①

$$R_B \cos 36^\circ 52' \, i - R_A \, i + R_B \sin 36^\circ 52' \, j - 100 \, j = 0$$

$$(R_B \cos 36^\circ 52' - R_A) \, i + (R_B \sin 36^\circ 52' - 100) \, j = 0$$

$$R_B \cos 36^\circ 52' - R_A = 0 \quad \text{——— (1)}$$

$$R_B \sin 36^\circ 52' - 100 = 0 \quad \text{——— (2)}$$

Solve for R_A & R_B

by using eqn ① & ②

For equilibrium condition of sphere ②

$$R_D i - R_B \cos 36°52' i + R_C j - R_B \sin 36°52' j - 100 j = 0$$

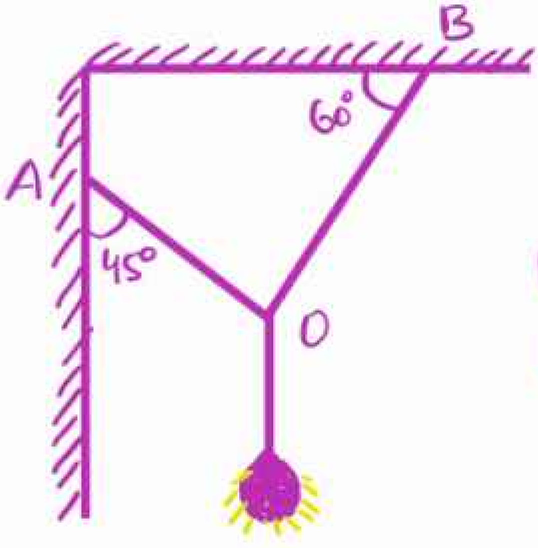
$$(R_D - R_B \cos 36°52') i + (R_C - R_B \sin 36°52' - 100) j = 0$$

$$R_D - R_B \cos 36°52' = 0 \qquad \text{③}$$

$$R_C - R_B \sin 36°52' - 100 = 0 \qquad \text{④}$$

Solve for R_C & R_D

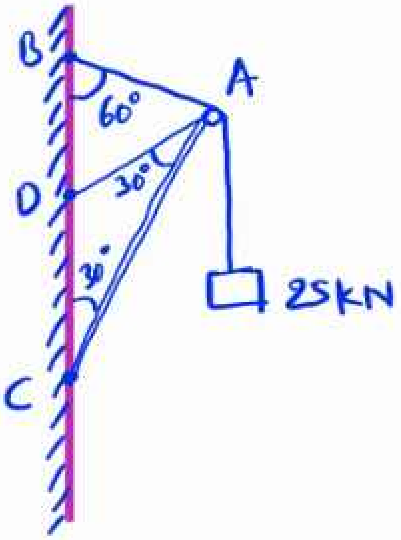
An electric light fixture weighting 150N is supported by two wires as shown in fig.. Determine the tension force developed in the wires.



$F_{OA} = 77.646$ N

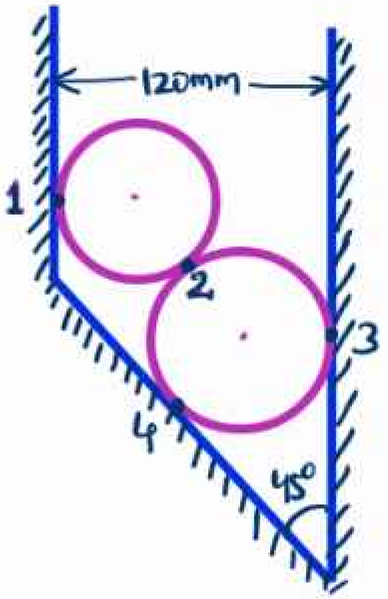
$F_{OB} = 109.808$ N

The frictionless pulley A shown in fig. is supported by two bars AB & AC which are hinged at B & C to a vertical wall. The flexiable cable DA hinged at D, goes over the pulley and supports a load of 25 KN at A. The angles between the various members are shown in fig. Determine the forces in the bars AB & AC. Neglect the size of pulley and treat it as frictionless.



$F_{AB} = 0$, $F_{AC} = 43.301$ KN

Two cylinders of diameter 100 mm and 50 mm, weighting 200N and 50N respectively are placed in a trough as shown in fig. Assuming all contact surface are smooth, find the reaction developed at contact surfaces 1, 2, 3 & 4.



$R_1 = 37.5\text{ N}$

$R_2 = 62.5\text{ N}$

$R_3 = 287.5\text{ N}$

$R_4 = 353.5\text{ N}$

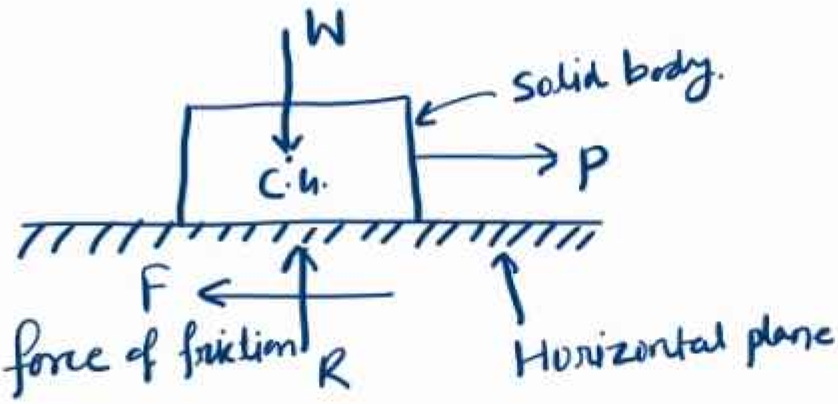
Friction: Introduction, limiting friction and impeding motion, Coulomb's laws of dry friction, coefficient of friction, Cone of Static friction. (P-150)

Friction

When a body moves (or) tend to move over another body, a force opposing the motion develops at the contact surfaces. This force which opposes the movement (or) the tendency of movement is called frictional force.

Coefficient of friction and angle of friction

For defining the terms like coefficient of friction (μ) and angle of friction (φ), consider a solid body placed on a horizontal plane surface.



where.

W = Weight of body acting through C.G. downward

R = Normal reaction of body action through C.G. upward

P = Force acting on the body through C.G. and parallel to the horizontal surface.

Resolving the surfaces on the body vertically and horizontally.

$$R = W$$

$$F = P$$

If the magnitude of P is further increased the body will start moving. The force of friction, acting on the body when the body is moving, is called kinetic friction.

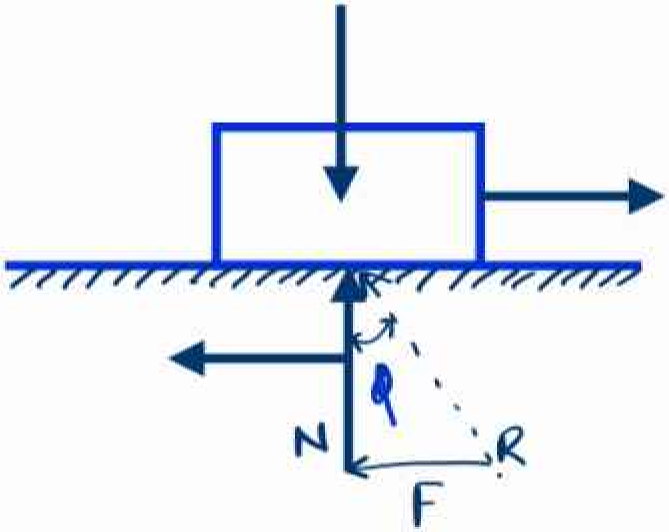
Coefficient of friction (μ)

It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies.

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$

$$F = \mu R$$

Limiting friction



Consider a block subjected to pull P. Let F be the frictional force developed and N the normal reaction.

Thus, at the contact surface, the reactions are F & N. They can be combined graphically to get the reaction R which acts at angle θ to normal reaction.

Angle of friction

As frictional force increases, the angle φ increases and it can reach maximum value α when limiting value of friction is reached.

$$\tan\alpha = \frac{F}{N} = \mu$$

and value of α is called Angle of limiting friction.

Angle of friction (ϕ)

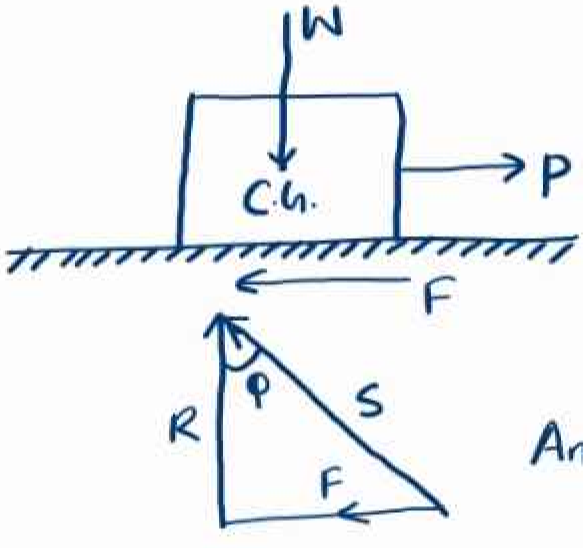
It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction (R).

Symbol $= \phi$

let, S = Resultant of the normal reaction (R) and limiting force of friction (F)

angle of friction $= \phi =$ Angle between S and R.

$$\tan\phi = \frac{F}{R} = \frac{\mu R}{R} = \mu = \text{coefficient of friction}$$



Angle of friction.

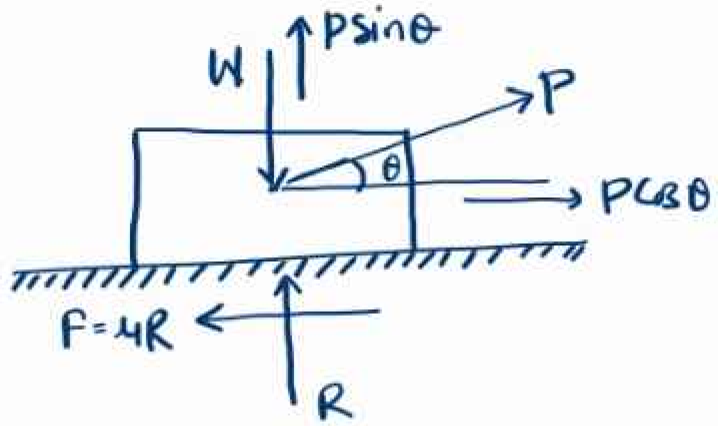
A block of weight W is placed on a rough horizontal plane surface as shown in figure and force P is applied at an angle θ with the horizontal such that the block just tend to move.

Let R = Normal reaction

μ = coefficient of friction

F = Force of friction

$= \mu R$



The force P is resolved in two components i.e. $P\cos\theta$ in horizontal direction and $P\sin\theta$ in the vertical direction.

Resolving forces on the block horizontally

$$F = P\cos\theta$$

$$\mu R = P\cos\theta \quad \text{——— (1)}$$

Resolving forces on the block vertically

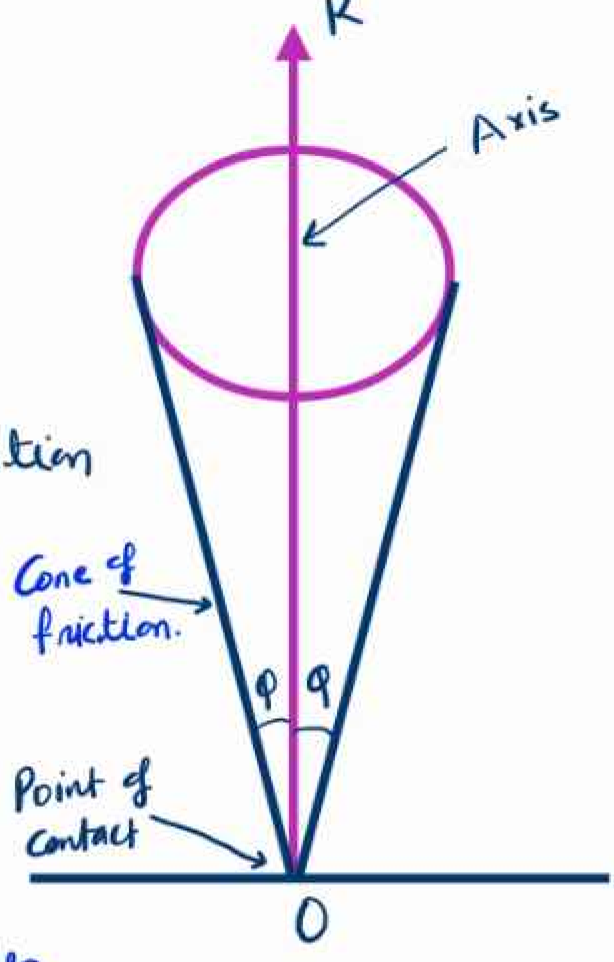
$$R + P\sin\theta = W$$

$$R = W - P\sin\theta \quad \text{------} \quad (2)$$

From eqⁿ (2), it is clear that normal is not equal to the weight of the block.

Cone of friction

It is defined as the right circular cone with vertex at the point of contact of two bodies (or surfaces), axis in the direction of normal reaction (R) and semi-vertical angle equal to angle of friction (φ).



Cone of friction

O = Point of contact between two bodies.

R = Normal reaction and also axis of cone of friction

φ = Angle of friction.

Type of friction

(1) Static and dynamic friction

(2) Wet and dry friction

Static and dynamic friction:

If the two surfaces, which are in contact, are at rest, the force experienced by on surface is called static friction.

If one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction.

Wet and Dry friction

If two surfaces, which are in contact, lubrication (oil or gas) is used, the friction that exists between two surfaces is known wet friction.

But, if no lubrication is used, then the friction between two surfaces is called Dry friction (or) solid friction.

Coulomb's law of friction

The friction, that exists between two surfaces which are not lubricated, is known as solid friction. The two surfaces may be at rest (or) one of the surface is moving and other surface is at rest.

(1) The force of friction always acts in a direction opposite to that in which the body tends to move.

(2) Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.

(3) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.

(4) The force of friction depends upon the roughness/smoothness of the surface.

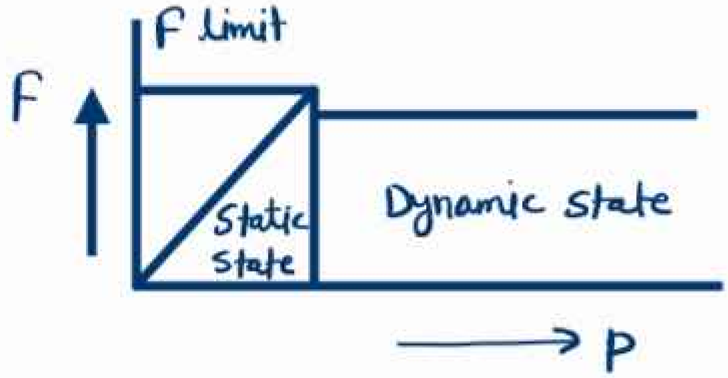
(5) The force of friction is independent of the area of contact between the two surfaces.

(6) After the body starts moving, the dynamic friction comes into play, the magnitude of which is less then that of limiting friction and it bears a constant ratio with normal force. This

ratio is called coefficient of dynamic friction.

Equilibrium analysis of simple system with sliding friction

When a tangential force P tend to move (or) moves a body over another body, according to Colomb's laws of motion, the frictional force F, developed between the two bodies is having variation with tangential force P.



If coefficient of static friction is μ, then $F = \mu N$ is the static state where N is normal reaction.

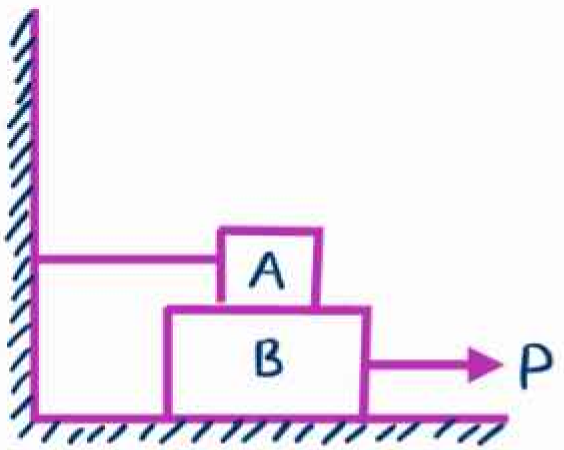
When motion is impending, $F = F_{limiting}$

Hence, $\boxed{F_{limiting} = \mu N}$

Angle of Repose.

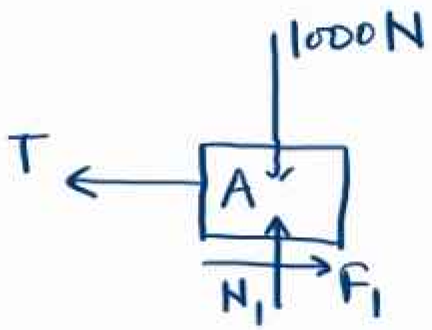
It is well known that

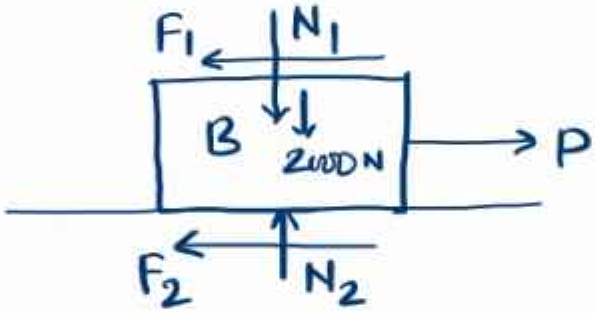
Block A weighting 1000N rests over block B which weights 2000N as shown in fig. Block A is tied to wall with a horizontal string. If the coefficient of friction between A and B is $\frac{1}{4}$ and between B and the floor is $\frac{1}{3}$, what should be the value of P to move the block B if (a) P is horizontal? (b) P acts 30° upwards to horizontal?



Soln.

(a)





For block A

$$\frac{F_1}{N_1} = \mu = \frac{1}{4}$$

$\Sigma V = 0$

$N_1 = 1000\,N$

$$F_1 = \frac{N_1}{4}$$

$\Sigma H = 0$

$$F_1 = \frac{1000}{4} = 250\,N$$

$F_1 = T$

$T = 250\,N$

For block B

$\Sigma V = 0$

$N_2 = N_1 + 2000 = 1000 + 2000 = 3000\,N$

$N_2 = 3000\,N$

$$\frac{F_2}{N_2} = \mu_2$$

$$\frac{F_2}{N_2} = \frac{1}{3} \Rightarrow F_2 = \frac{1}{3} N_2 = \frac{1}{3} \times 3000$$

$$= 1000\,N$$

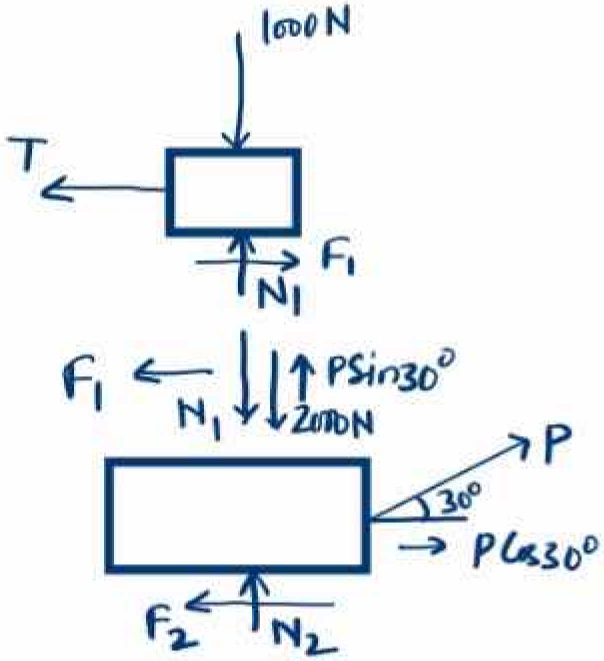
$F_2 = 1000\,N$

$\Sigma H = 0$

$F_1 + F_2 = P$

$P = 250 + 1000 = 1250\,N$

(b) When P is inclined



$$\Sigma V = 0$$

$$N_2 + P\sin 30^\circ - N_1 - 2000 = 0$$

$$N_2 + \frac{P}{2} - N_1 - 2000 = 0$$

$$N_2 = 2000 + 1000 - 0.5P = 3000 - 0.5P$$

From law of friction

$$F_2 = \frac{1}{3} N_2$$

$$F_2 = \frac{1}{3}(3000 - 0.5P)$$

$$\Sigma H = 0$$

$$F_1 + F_2 = P\cos 30^\circ$$

$$F_1 + F_2 - P\cos 30^\circ = 0$$

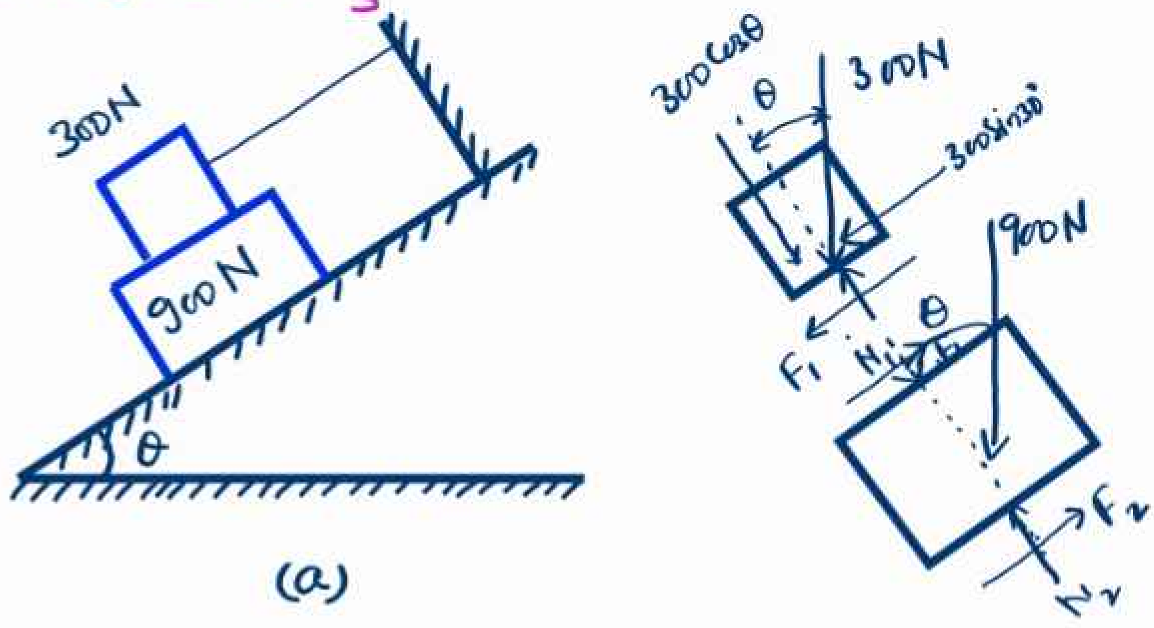
$$250 + \frac{1}{3}(3000 - 0.5P) - P\cos 30^\circ = 0$$

$$250 + 1000 - \frac{0.5}{3}P - \frac{P\sqrt{3}}{2}$$

$$1.0327\,P = 1250$$

$$P = \frac{1250}{1.0327} = 1210.42\ N$$

What should be the value of θ in fig. which will make the motion of 900N block down the plane to impend? The coefficient of friction for all contact surfaces is $\frac{1}{3}$.



Sol^n. 900N block is on the verge of moving downward. Hence frictional forces F_1 and F_2 act upon the plane on 900N block.

For 300N block

$\sum$ Forces normal to plane $=0$

$$N_1 - 300\cos\theta = 0$$

$$N_1 = 300\cos\theta$$

From law of friction

$$\frac{F_1}{N_1} = \frac{1}{3}$$

$$F_1 = \frac{1}{3}N_1 = 100\cos\theta \quad \text{——— (1)}$$

For 900N block

$\sum$ Forces normal to the plane $=0$

$$N_2 - N_1 - 900\cos\theta = 0$$

$$N_2 = N_1 + 900\cos\theta \quad \text{——— (2)}$$

Substituting the value of N_1 from (1)

$$N_2 = 300\cos\theta + 900\cos\theta$$

$$N_2 = 1200\cos\theta$$

from law of friction

$$F_2 = \frac{1}{3} N_2 = \frac{1}{3} \times 1200 \cos\theta$$

$$F_2 = 400 \cos\theta$$

Σ Forces parallel to the plane $= 0$

$$F_1 + F_2 - 900 \sin\theta = 0$$

$$100 \cos\theta + 400 \cos\theta = 900 \sin\theta$$

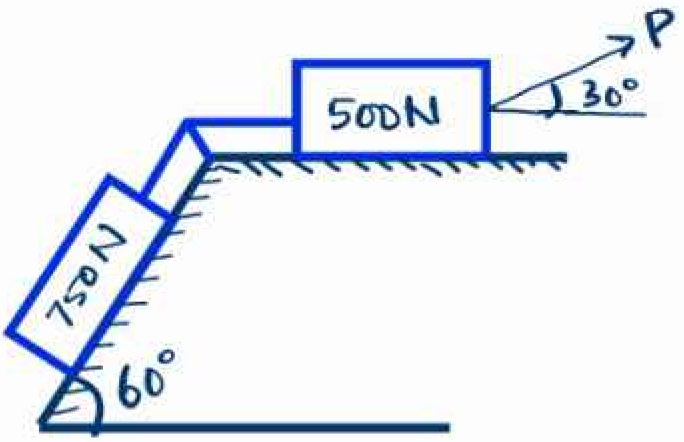
$$500 \cos\theta = 900 \sin\theta$$

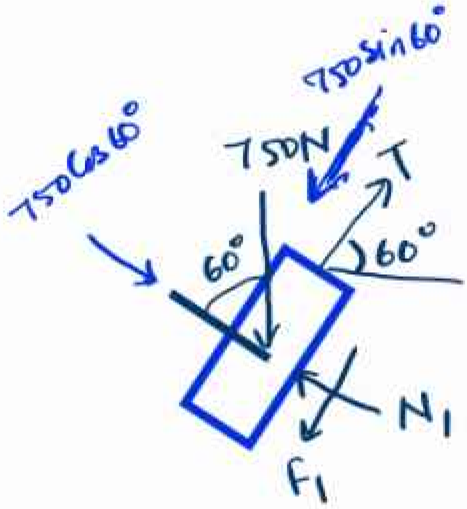
$$\tan\theta = \frac{500}{900} = \frac{5}{9}$$

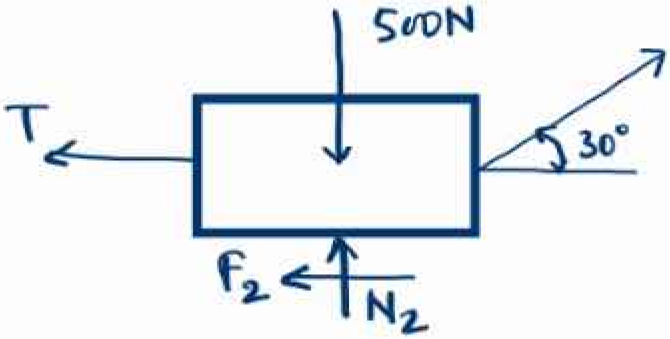
$$\theta = \tan^{-1}(5/9)$$

$$\boxed{\theta = 29^\circ 3'}$$

What is the value of P in the system shown in fig. to cause the motion to impend? Assume the pulley is smooth and coefficient of friction b/w the other contact surface is 0.20.







Considering 750N block

$\sum$ Force normal to the plane $= 0$

$$N_1 - 750 \cos 60° = 0$$

$$N_1 = 375\ N$$

Since the motion is impending, from law of friction

$$F_1 = \mu N_1$$

$$F_1 = 0.2 \times 375 = 75\ N$$

$\sum$ Forces parallel to the plane $= 0$

$$T - F_1 - 750 \sin 60° = 0$$

$$T = F_1 + 750 \sin 60°$$

$$T = 75 + 750 \sin 60° = 724.52\ N$$

Considering 500N body

$$\Sigma V = 0$$

$$N_2 - 500 + P\sin 30^\circ = 0$$

$$N_2 + 0.5P = 500$$

From law of friction

$$F_2 = 0.2 N_2$$

$$= 0.2(500 - 0.5P)$$

$$= 100 - 0.1P$$

$$\Sigma H = 0$$

$$P\cos 30^\circ - T - F_2 = 0$$

$$P\cos 30^\circ - 724.52 - 100 + 0.1P = 0$$

$$P = 853.2\,N$$

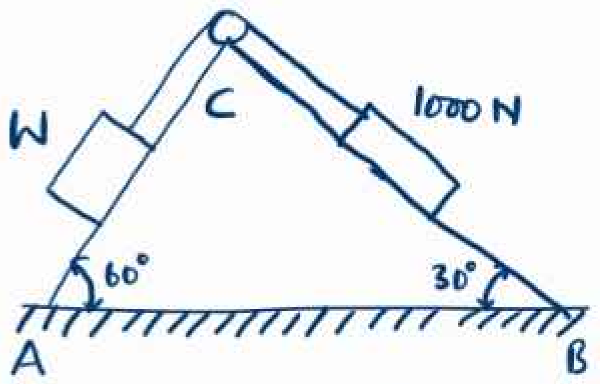
Absent - 17/03/25

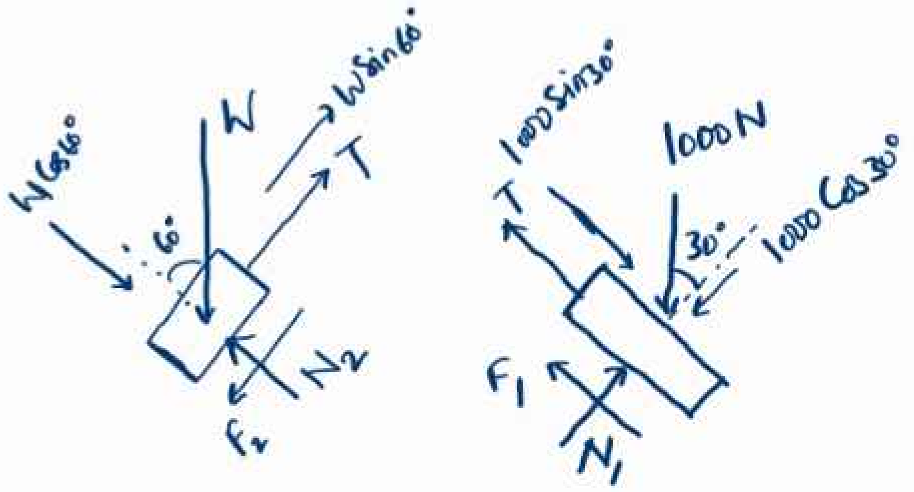
Absent

5, 6, 8, 16, 17, 22, 25,

31, 32, 33, 35, 37, 38, 50, 51,

Two identical planes AC and BC inclined at 60° and 30° to the horizontal, meet at C. A load of 1000N rests on the inclined plane BC and is tied by a rope passing over a pulley to a block weighting W Newtons and resting on the plane AC. If the coefficient of friction between the load and the plane BC is 0.28 and that between the block and the plane AC is 0.20, find the least and the greatest value of W for equilibrium of the system.





Consider 1000N block $(\mu = 0.28)$

Σ Forces normal to the plane $= 0$

$$N_1 = 1000 \cos 30^\circ = 866.03 N$$

From law of friction,

$$\frac{F_1}{N_1} = 0.28$$

$$F_1 = 0.28 N_1$$

Σ Forces parallel to the plane $= 0$

$$-T - F_1 + 1000 \sin 30^\circ = 0$$

$$T + F_1 = 1000 \sin 30^\circ$$

$$T + 0.28 N_1 = 1000 \sin 30^\circ$$

$$T + 0.28 \times 866.03 = 1000 \sin 30^\circ$$

$$T = 1000 \sin 30^\circ - 0.28 \times 866.03$$

$$T = 257.51 N$$

Now consider block with weigh W

$\Sigma V = 0$

$$N_2 = W\cos 60^\circ = 0.5W$$

$$F_2 = 0.2N_2 = 0.2 \times 0.5W = 0.1W$$

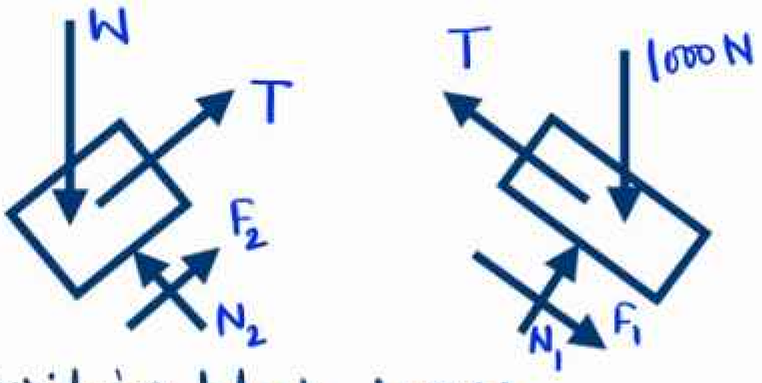
$\Sigma H = 0$

$$F_2 + W\sin 60^\circ = T$$

$$W = 266.57N$$

For greatest value of W, the 1000N block is on the verge of moving up.

$\therefore$ For such case, the free body diagram



Considering block 1000 N.

$$N_1 = 866.03\,N$$

$$F_1 = 242.49\,N.$$

$$T = 1000\sin 30^\circ + F_1 = 742.49\,N$$

Considering block W

$$N_2 = W\cos 60^\circ = 0.5W$$

$$F_2 = 0.2N_2 = 0.1W$$

and $$W\sin 60^\circ - F_2 = T$$

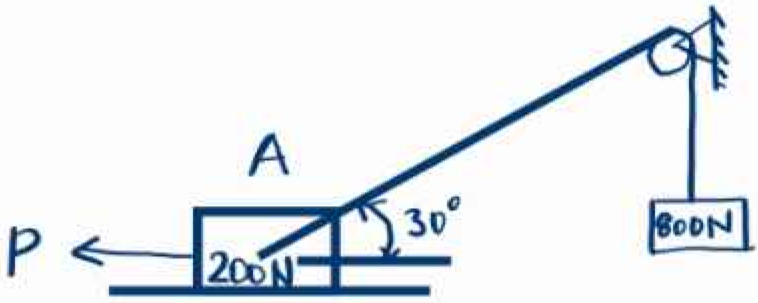
$$W(\sin 60^\circ - 0.1) = 742.49$$

$$W = 969.28\,N$$

The Block A as shown in figure weights 2000N. The chord attached to a passes over a frictionless pulley and supports a weight equal to 800N. The value of coefficient of friction between A and the horizontal plane is 0.35. Solve for the horizontal force P.

(i) If motion is impending towards the left.

(ii) If the motion is impending towards right.



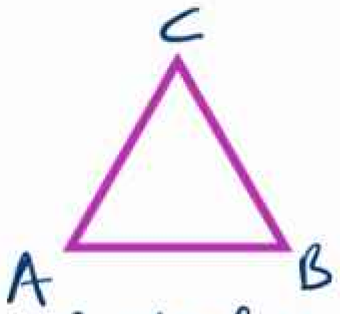
Ans. (i) $P = 1252.82$ N

(ii) $P = 132.82$ N

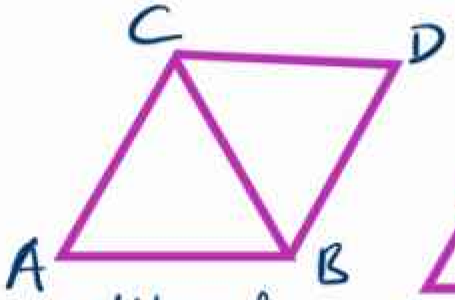
Plane frame (or) Truss structure

A plane frame structure (or) truss is made up of a number of members connected to one another at several joints and is used to support external loads.

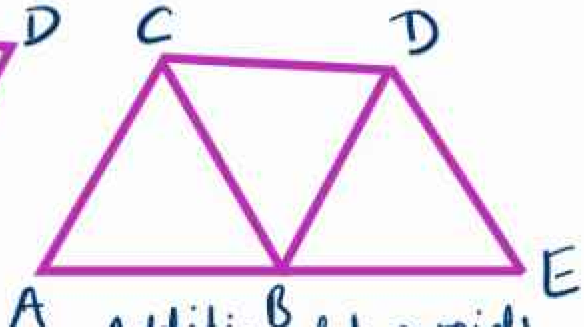
Perfect frame:



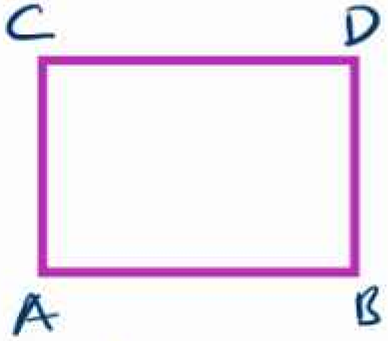
Basic (perfect) frame



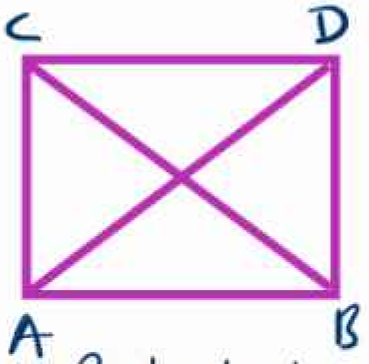
Addition of one joint



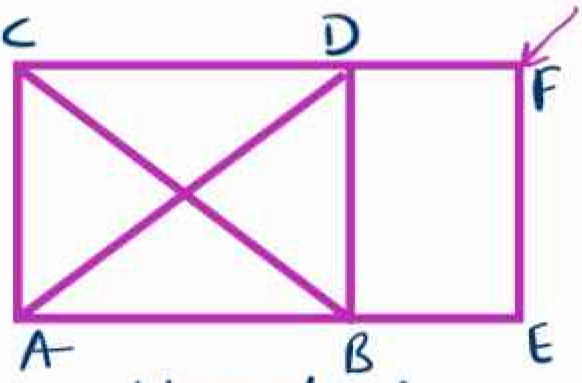
Addition of two joints



Deficient fram



Redundant frame



Non perfect frame

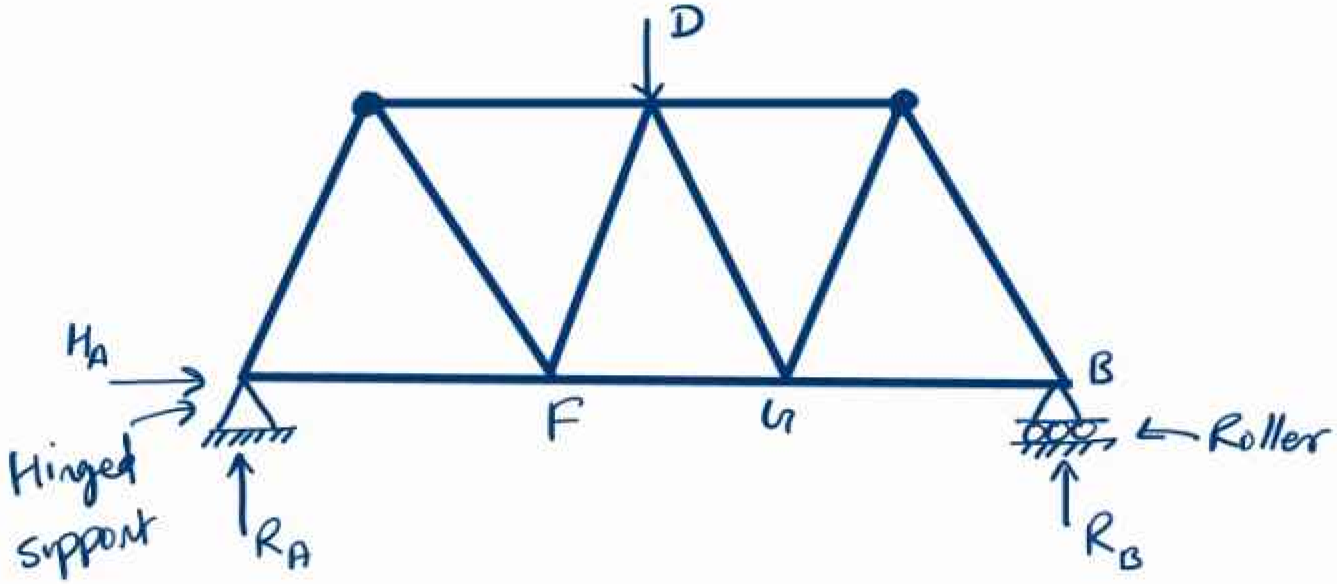
Assumptions made in finding out the axial forces in a frame are:—

(i) The frame is a perfect frame.

(ii) The frame carries load at joints.

(iii) All the members are pin jointed.

Reaction of supports of a frame:



Analysis of frame

Analysis of frame consist of

(i) Determinations of reaction at support

(ii) Determination of axial forces in the members of the frame.

* The reactions are determined by the condition that the applied load system and the induced reactions at the support from a system in equilibrium.
* The forces in the members of the frame are determine by the condition that every joint should in equilibrium, and so, the forces acting at every joint should form a system in equilibrium.

Methods to analyse framed structure.

(i) Method of joints

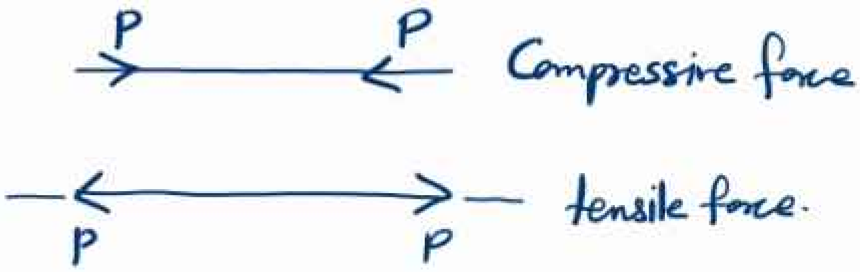
(ii) Method of sections

(iii) Graphical method

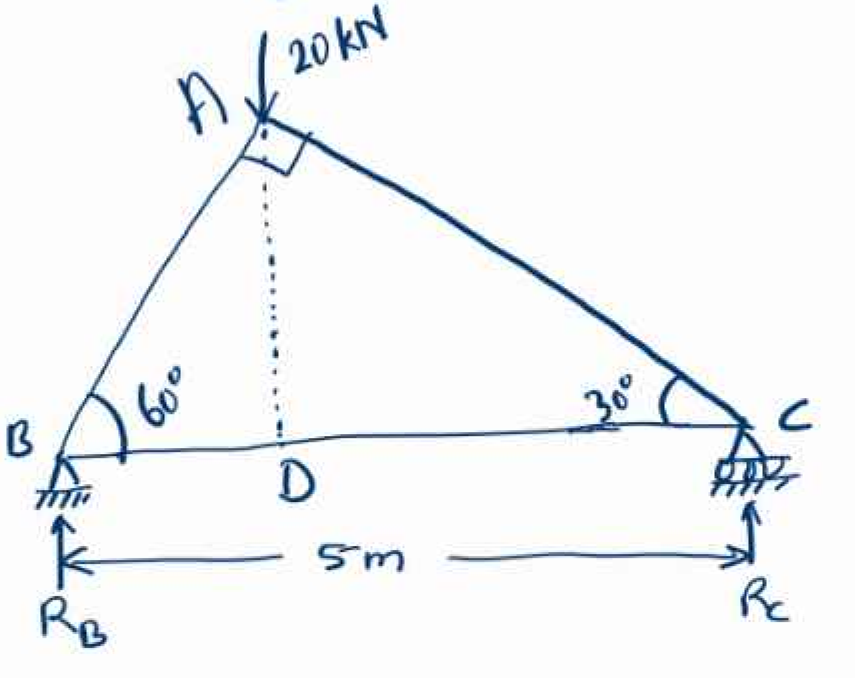
(i) Method of joints

In this method, after determining the reactions at the support, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero

The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if member is pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.



Find the forces in the members AB, AC and BC of the truss shown in fig.



First determine $\underline{R_B \& R_C}$

$$\cos 60^\circ = \frac{AB}{BC} = \frac{AB}{5}$$

$$AB = 5\cos 60^\circ = 2.5\,m$$

$$\cos 60^\circ = \frac{BD}{AB} = \frac{BD}{2.5}$$

$$BD = 2.5 \times \cos 60^\circ = 2.5 \times \tfrac{1}{2} = 1.25\,m$$

<u>Now take, moment about B</u>

$$R_C \times 5 - 20 \times 1.25 = 0$$

$$5R_C = 25$$

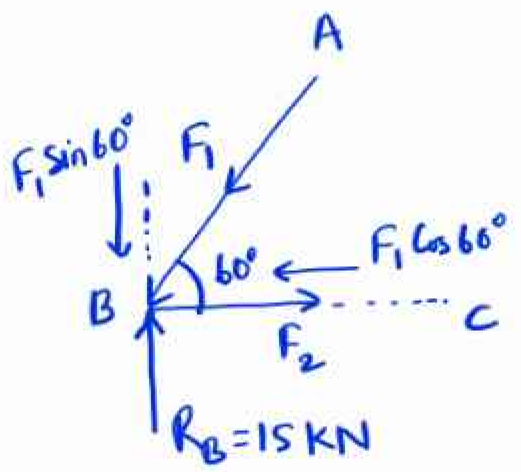
$$R_C = 5\,KN$$

$$R_B + R_C = 20\,KN$$

$$R_B = 20 - 5 = 15\,KN.$$

Now consider the equilibrium of various points.

<u>Consider point B</u>



F_1 = Force in member AB

F_2 = Force in member BC

$\Sigma V = 0$

$-F_1 \sin 60° + 15 = 0$

$F_1 \sin 60° = 15$

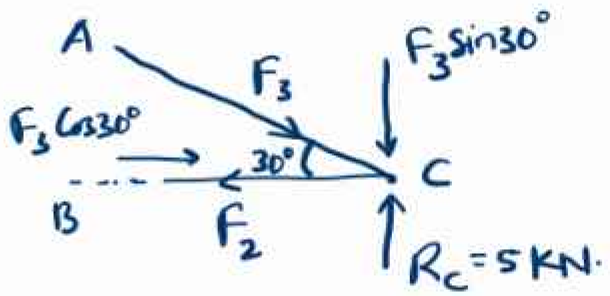
$F_1 = \dfrac{15}{\sin 60°} = 17.32\ KN$ (Compression)

$\Sigma H = 0$

$F_2 - F_1 \cos 60° = 0$

$F_2 = F_1 \cos 60° = 17.32 \times \cos 60° = 8.66\ KN$ (tensile)

Joint C



Let

F_3 = Force in member AC

F_2 = Force in member BC

$\Sigma V = 0$

$R_C - F_3 \sin 30^\circ = 0$

$5 - F_3 \sin 30^\circ = 0$

$F_3 \sin 30^\circ = 5$

$F_3 = \frac{5}{\sin 30^\circ} = 10\ kN$ (Compressive)

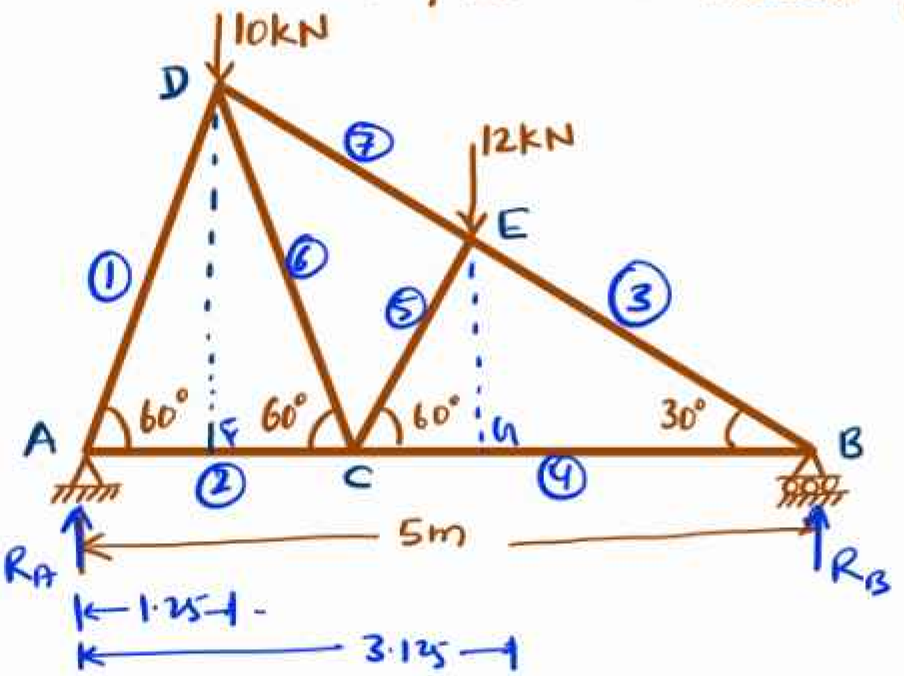
$\Sigma H = 0$

$-F_2 + F_3 \cos 30^\circ = 0$

$F_3 \cos 30^\circ = F_2$

$F_3 = \frac{8.66}{\cos 30^\circ} = \frac{8.66 \times 2}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = \frac{17.32}{1.732} = 10 kN$

\# A truss of span 5m is loaded as shown in fig. Find the reactions and forces in the members of the truss.



Soln.

From triangle ABD.

$$AD = AB\cos 60^\circ = 5 \times \frac{1}{2} = 2.5\,m$$

The distance of line of action of vertical force 10 KN.

$$\cos 60^\circ = \frac{AF}{AD}$$

$$AF = 2.5 \times \cos 60^\circ = 1.25\,m$$

Here, $AC = AD = 2.5\,m.$

$$\therefore \quad CB = AB - AC = 5 - 2.5 = 2.5\,m$$

To find the distance BE

$$\sin 60^\circ = \frac{BE}{BC} = \frac{BE}{2.5}$$

$$BE = 2.5 \times \sin 60^\circ = 2.165\,m$$

The distance of line of action of vertical load 12 KN.

$$\cos 30^\circ = \frac{BG}{BE}$$

$$BG = 2.165 \times \frac{\sqrt{3}}{2} = 1.875\,m$$

$$AG = AB - BG = 5 - 1.875 = 3.125\,m$$

Now, take moment about A

$$R_B \times 5 - 12 \times 3.125 - 10 \times 1.25 = 0$$

$$5R_B = 12 \times 3.125 + 10 \times 1.25 = 50$$

$$R_B = 10 \text{ kN}$$

$$R_A + R_B = 22$$

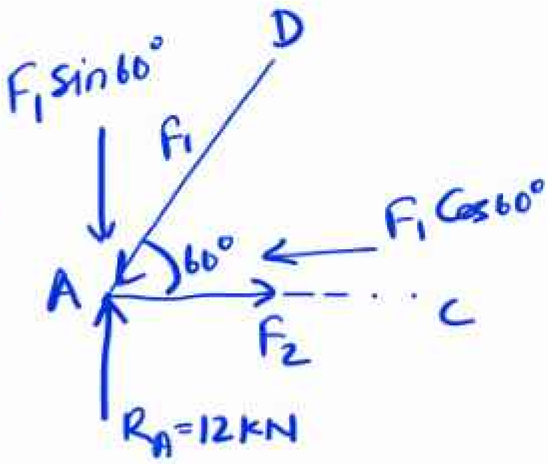
$$R_A = 22 - 10 = 12 \text{ kN.}$$

Now consider Equilibrium of various joints

Joint A

F_1 = Force in member AD

F_2 = Force in member AC



$\Sigma V = 0$

$$R_A - F_1 \sin 60° = 0$$

$$12 - F_1 \sin 60° = 0$$

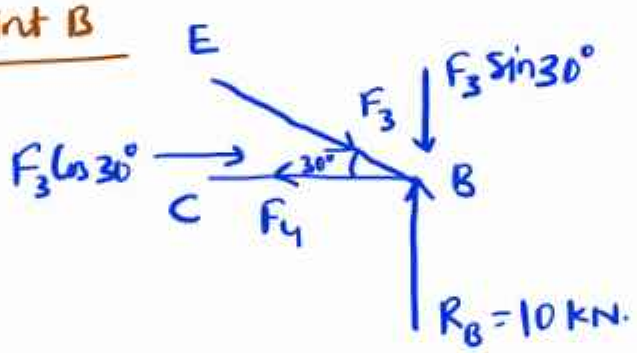
$$F_1 = \frac{12}{\sin 60°} = 13.856 \text{ kN.}$$

$\Sigma H = 0$

$$F_2 - F_1 \cos 60^\circ = 0$$

$$F_2 = 13.856 \times \cos 60^\circ = 6.928 \text{ KN.}$$

Joint B



F_3 = force in member BE

F_4 = force in member BC

$\Sigma V = 0$

$$F_3 \sin 30^\circ = 10$$

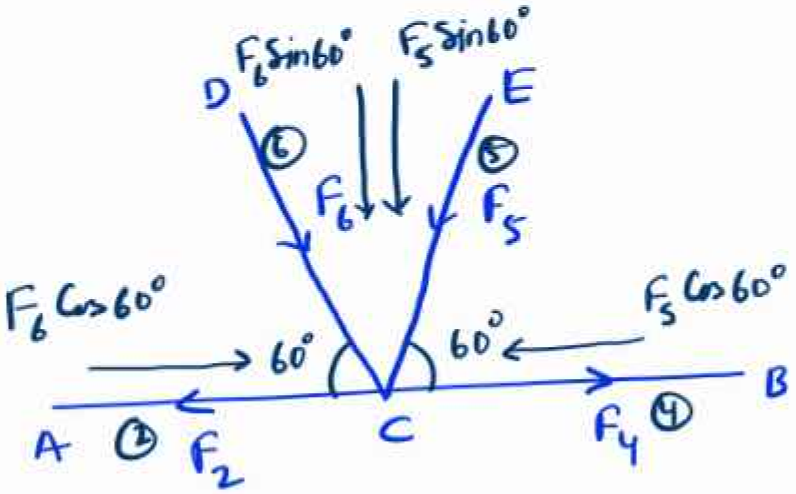
$$F_3 = \frac{10}{\sin 30^\circ} = 20 \text{ KN}$$

$\Sigma H = 0$

$$F_4 = F_3 \cos 30^\circ$$

$$F_4 = \frac{10}{\sin 30^\circ} \times \cos 30^\circ = 17.32 \text{ KN}$$

Joint C



$\underline{\Sigma V = 0}$

$$-F_6 \sin 60° - F_5 \sin 60° = 0$$

$$F_5 = -F_6$$

$\Sigma H = 0$

$$F_4 - F_2 + F_6 \cos 60° - F_5 \cos 60° = 0$$

$$17.32 - 6.928 - F_5 \cos 60° - F_5 \cos 60° = 0$$

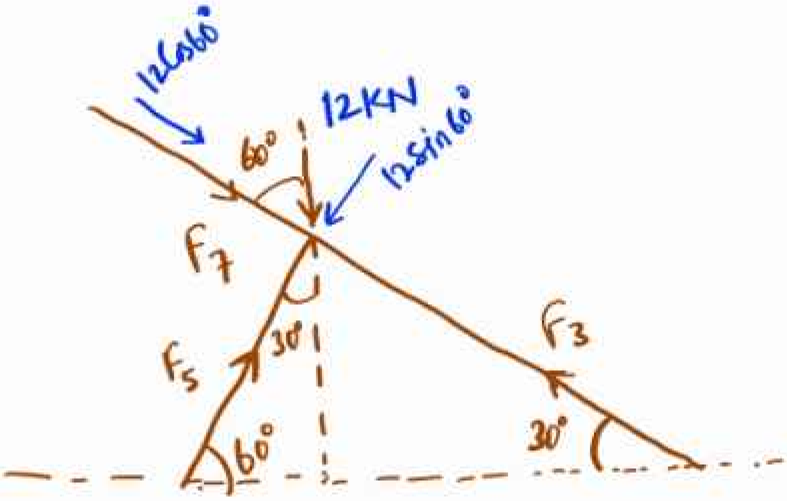
$$F_5 (\cos 60° + \cos 60°) = 17.32 - 6.928$$

$$F_5 = \frac{17.32 - 6.928}{2 \cos 60°} = \frac{10.392}{2 \times 1/2} = 10.392 \text{ kN}$$

(Compressive)

$$F_6 = -10.392 \text{ kN}. \text{ (Tensile)}$$

Joint E



F_7 = Force in member ED.

Let F_7 is acting as shown.

$$F_7 + 12\cos 60^\circ = F_3$$

$$F_7 = 20 - 12 \times \tfrac{1}{2} = 14\ \text{kN}.\ \text{(Compressive)}$$